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Filières SM-SMI / Module M_6 / Analyse 2
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FORMULAIRE

LES FONCTIONS CIRCULAIRES ET LEURS FONCTIONS RECIPROQUES

$\cos^2 a + \sin^2 b = 1$

$\cos(a+b) = \cos a.\cos b - \sin a \sin b$ $\qquad \cos(a-b) = \cos a.\cos b + \sin a \sin b$

$\sin\,(a+b) = \sin a.\cos b\ + \sin b.\cos a$ $\qquad \sin\,(a-b) = \sin a.\cos b\ - \sin b.\cos a$

$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a.\tan b}$ $\qquad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a.\tan b}$

$\cos p + \cos q = 2.\cos\frac{p+q}{2}.\cos\frac{p-q}{2}$ $\qquad \cos p - \cos q = -2.\sin\frac{p+q}{2}.\sin\frac{p-q}{2}$

$\sin p\ + \sin q\ = 2.\sin\frac{p+q}{2}.\cos\frac{p-q}{2}$ $\qquad \sin p - \sin q = 2.\sin\frac{p-q}{2}.\cos\frac{p+q}{2}$

$\cos p.\cos q = \frac{1}{2}[\cos(p+q) + \cos(p-q)]$ $\qquad \sin p.\sin q = \frac{1}{2}[\cos(p-q) - \cos(p+q)]$

$\sin p.\cos q = \frac{1}{2}[\sin(p+q) + \sin(p-q)]$

Changement de variable :

Soit $t = \tan\frac{x}{2}$, on a : $\cos x = \frac{1-t^2}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$, $\tan x = \frac{2t}{1-t^2}$

Dérivées :

$\cos' x = -\sin x$, $\sin' x = \cos x$, $\tan' x = 1 + \tan^2 x$, $\cot an' x = -1 - \cot an^2 x$

$Arc\cos' x = \frac{-1}{\sqrt{1-x^2}}$ ($|x| < 1$) , $Arc\sin' x = \frac{1}{\sqrt{1-x^2}}$ ($|x| < 1$)

$Arc\tan' x = \frac{1}{1+x^2}$, $Arcco\tan' x = \frac{-1}{1+x^2}$

LES FONCTIONS HYPERBOLIQUES ET LEURS FONCTIONS RECIPROQUES

$ch(x) = \frac{e^x + e^{-x}}{2}$, $sh(x) = \frac{e^x - e^{-x}}{2}$, $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$Argchx = \ln(x + \sqrt{x^2 - 1})$ $(x \geq 1)$, $\qquad Argshx = \ln(x + \sqrt{x^2 + 1})$

$Argthx = \frac{1}{2}\ln(\frac{1+x}{1-x})$ $(|x| < 1)$, $\qquad Arg\coth x = \frac{1}{2}\ln(\frac{1+x}{x-1})$ $(|x| > 1)$

$ch^2 a - sh^2 b = 1$

$ch(a+b) = cha.chb + sha.shb$ $\qquad ch(a-b) = cha.chb - sha.shb$

$sh\,(a+b) = sha.chb\ + shb.cha$ $\qquad sh\,(a-b) = sha.chb\ - shb.cha$

$\tanh(a+b) = \frac{\tanh a + \tanh b}{1 + \tanh a.\tanh b}$ $\qquad \tanh(a-b) = \frac{\tanh a - \tanh b}{1 - \tanh a.\tanh b}$

$chp + chq = 2.ch\frac{p+q}{2}.ch\frac{p-q}{2}$ $\qquad chp - chq = 2.sh\frac{p+q}{2}.sh\frac{p-q}{2}$

$chp.chq = \frac{1}{2}[ch(p+q) + ch(p-q)]$ $\qquad shp.shq = \frac{1}{2}[ch(p+q) - ch(p-q)]$

$shp.chq = \frac{1}{2}[sh(p+q) + sh(p-q)]$

Changement de variable :

Soit $t = \tanh\frac{x}{2}$, on a : $chx = \frac{1+t^2}{1-t^2}$, $shx = \frac{2t}{1-t^2}$, $\tanh x = \frac{2t}{1+t^2}$

Dérivées :

$ch' x = shx$, $sh' x = chx$, $\tanh' x = 1 - \tanh^2 x$, $\cot anh' x = 1 - \cot anh^2 x$

$Argch' x = \frac{1}{\sqrt{x^2-1}}$ ($x > 1$) , $Argsh' x = \frac{1}{\sqrt{1+x^2}}$

$Arg\tanh' x = \frac{1}{1-x^2}$ ($|x| < 1$) , $Argco\tanh' x = \frac{1}{1-x^2}$ ($|x| > 1$)