



On Black Objects in Type IIA Superstring Theory on Calabi-Yau Manifolds

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Abstract

The compactification of 10D type IIA superstrings on n -dimensional Calabi-Yau manifolds is discussed. In particular, a conjecture is given for type IIA p -dimensional black object attractors corresponding to $AdS_{p+2} \times S^{8-2n-p}$ horizon geometries.

Keywords: Type IIA Superstring, Calabi-Yau manifolds, Black Objects.

I. INTRODUCTION

Recently, $N = 2$ four-dimensional (4D) black hole (BH) physics has attracted much attention and been investigated using different connections with string theory, branes and compactifications. One nice result in this context is the Ooguri, Strominger and Vafa conjecture relating the microstates counting of 4D $N = 2$ BPS BH in type II superstrings on Calabi-Yau (CY) threefolds to the topological string partition function on the same geometry¹. This conjecture has been extended in^{2,3} to open topological strings, which capture information on BPS states on D-branes wrapped on Lagrangian submanifolds of CY 3-folds, and to various toric CY threefolds⁴.

On the other hand, the 4D $N = 2$ BH exhibits an interesting phenomenon called the attractor mechanism⁵⁻¹⁹ which is the topic of interest here. At the horizon, the moduli scalar fields take fixed values given in terms of the BH charges. This can be understood in terms of an effective potential which depends on the charges and the moduli²⁰. Extremising the potential with respect to the moduli, the minimum gives such fixed values.

The 4D $N = 2$ attractors have been extended to other supergravities in 4D as well as to higher dimensions²¹⁻²⁸. In five dimensions, for instance, an M-theory realisation of such BH attractors has been studied²³. The corresponding low-energy theory also has $N = 2$ supersymmetry involving real special geometry. It turns out that the 4D $N = 2$ black attractors are connected to the 5D attractors²⁹. More recently, the black object attractors in 6D and 7D have been given in²⁴. In particular, the extremal BPS and non BPS black attractors in $N = 2$ supergravity, embedded in type IIA superstring theory and 11D M-theory on K3, respectively, have been studied. The attractor mechanism equations and their solutions have been treated using the criticality condition of the attractor potential of such black objects.

Here we consider type IIA superstrings on CY n -folds and discuss the corresponding p -dimensional black object attractors. In this scenario, the near-horizon geometries are given by products of

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AdS_{p+2} and $(8 - 2n - p)$ -dimensional real spheres S^{8-2n-p} . In the context of the type IIA attractor mechanism on CY n -folds, we conjecture that:

$$\begin{array}{lll}
 p = 0 & \text{black hole charges fix} & \rightarrow \text{geometric moduli} \\
 p = 3 - n & \text{Dyonic black brane charges fix} & \rightarrow \text{the dilaton} \\
 p \neq 0, 3 - n & \text{black brane charges fix} & \rightarrow \mathbf{R-R} \text{ stringy moduli.}
 \end{array}$$

II. TYPE IIA SUPERSTRING ON CY SPACES AND BLACK BRANE CONFIGURATIONS

We start with some general observation about Type IIA superstrings on CY manifolds and black brane configurations in the compactified theory. It is noted that an n -dimensional CY manifold is defined by the following conditions: Complex, Kähler, and existence of a global nonvanishing holomorphic n -form. Equivalently, it is a Kähler manifold with a vanishing first Chern class $c_1 = 0$ ($SU(n)$ Holonomy group). After compactification, it preserves only $\frac{1}{2^{n-1}}$ of the initial supercharges. Note that each manifold involves a Hodge diagram playing a crucial role in the determination of the string theory spectrum in lower dimensions³⁰⁻³². For later reference, we table the Hodge diagrams for $n = 1, 2, 3$ corresponding to T^2 torus, K3 surface and CY threefolds, respectively

$n = 1$	$ \begin{array}{ccc} & h^{0,0} & \\ h^{1,0} & & h^{0,1} \\ & h^{1,1} & \end{array} $	$ \begin{array}{ccc} & & 1 \\ 1 & & 1 \\ & & 1 \end{array} $
$n = 2$	$ \begin{array}{ccccc} & & h^{0,0} & & \\ & h^{1,0} & & h^{0,1} & \\ h^{2,0} & & h^{1,1} & & h^{0,2} \\ & h^{2,1} & & h^{1,2} & \\ & & h^{2,2} & & \end{array} $	$ \begin{array}{ccc} & & 1 \\ 0 & & 0 \\ 1 & 20 & 1 \\ 0 & & 0 \\ & & 1 \end{array} $
$n = 3$	$ \begin{array}{ccccccc} & & & h^{0,0} & & & \\ & h^{1,0} & & h^{0,1} & & & \\ h^{2,0} & & h^{1,1} & & h^{0,2} & & \\ h^{3,0} & h^{2,1} & & h^{1,2} & & h^{0,3} & \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ & & h^{3,2} & & h^{2,3} & & \\ & & & h^{3,3} & & & \end{array} $	$ \begin{array}{ccccccc} & & & h^{0,0} & & & \\ & 0 & & 0 & & & \\ 0 & & h^{1,1} & & 0 & & \\ 1 & h^{2,1} & & h^{1,2} & & 1 & \\ 0 & & h^{1,1} & & 0 & & \\ & 0 & & 0 & & & \\ & & & 1 & & & \end{array} $

From this table, we can see that each diagram contains two central orthogonal lines, obtained by deleting the zeros. The vertical line represents parameters of the Kähler deformations and the horizontal one encodes the moduli of the complex structure of the CY n -folds. The number of parameters representing the Kähler deformations of the metric is fixed by $h^{1,1}$; while the number of the complex structure deformations is given by $h^{n-1,1}$. The mirror version is obtained by interchanging the vertical and horizontal lines.

It turns out that the hodge diagram of CY manifolds carry not only geometric information (Kähler and complex deformations) but also physical information. Indeed, the moduli space of CY Type IIA superstring compactification involves three different contributions depending on the entries of the Hodge diagrams. They are given by:

- The geometric deformations of the CY space including the antisymmetric B-field of the **NS-NS** sector. This involves the complex structure deformations, the complexified Kähler deformations or both.
- The dilaton defining the string coupling constant.
- The scalar moduli coming from **R-R** gauge fields on non trivial cycles of the CY spaces.

Note that in the case of K3 surface, the last contribution does not appear due the fact that $h^{1,0} = h^{1,2} = h^{0,1} = h^{2,1} = 0$. Then we have only two parts. Indeed, the 10D perturbative bosonic

III. CONJECTURE ON TYPE IIA BLACK OBJECTS ON CY MANIFOLDS

Before going ahead, let us recall some results obtained in this context. Consider type IIA superstring propagating on a CY 3-folds. This leads to a 4D $N = 2$ supergravity coupled to $U(1)^{h_{1,1}}$ abelian gauge symmetry. In this theory, the scalars belonging to the vector multiplets are given by the complexified Kähler moduli space. The complex structure deformation corresponding to 3-cycles and the **R-R** gauge fields contribute by scalar in the the hypermultiplets. However, this sector has no role in the study of the 4D BH attractors. Indeed, several studies of the 4D type IIA super attractor mechanism reveal that in the near horizon geometry only the complexified Kähler moduli can be fixed by the abelian BH charges⁵⁻¹⁹. This can be obtained by minimising the scalar effective potential of the 4D BH²⁰. This potential appears in the action of the $N = 2$ supergravity theory coupled to the Maxwell theory in which the abelian gauge vectors come from the reduction of the antisymmetric 3-form on 2-cycles of the CY 3-folds¹. The other moduli, corresponding either to the complex structure deformations or to the moduli coming from the **R-R** gauge fields on CY cycles, remain free and take arbitrary values in the near horizon limit of BH. We believe that this is due to the absence of higher-dimensional black objects in four dimensions². However, the situation in six dimensions is somewhat different. It is obtained by a compactification on the K3 complex surface. The latter has a mixed geometric moduli space involving both the complexified Kähler and the complex structure deformations. The corresponding attractor mechanism for the black objects has been studied in²⁴, see also²⁵⁻²⁷. Indeed, using a matrix formulation, the geometric moduli of the K3 surface including the **NS-NS** B-field are fixed by the abelian BH charges. However, the dilaton has been fixed by the black fundamental string charges (the electric and magnetic charges). Having discussed the known results, we now propose our conjecture for the CY attractor mechanism. The main idea is that the moduli of Type IIA superstrings on a CY manifold could be fixed by the charges of the black objects appearing in the compactified theory. Indeed, consider Type IIA superstring on an n -dimensional Calabi-Yau manifold. This compactification gives supergravity models with 2^{6-n} supercharges coupled to an abelian gauge symmetry. The near horizon of the corresponding the black objects is defined by the product of Ads spaces and spheres:

$$AdS_{p+2} \times S^{8-2n-p} \quad (\text{III.1})$$

The numbers n and p satisfy

$$2 \leq 8 - 2n - p \quad (\text{III.2})$$

The electric/ magnetic duality relating the p -dimensional electrical black objects to q -dimensional magnetic ones reads as

$$p + q = 6 - 2n \quad (\text{III.3})$$

This equation can be solved in different ways, here indicated by the values for (p, q) :

- $(p, q) = (0, 6 - 2n)$; describing an electrical charged BH.
- $(p, q) = (3 - n, 3 - n)$; describing Dyonic black branes.
- $(p, q) \neq (0, 6 - 2n)$ and $(p, q) \neq (3 - n, 3 - n)$; describing black objects like strings, membranes and higher-dimensional branes.

More precisely, we conjecture the following:

¹Note that the the parameters of the complex structure has been fixed in the context of type IIB superstrings using mirror symmetry

²This can be seen from the electric-magnetic duality equation.

- The BH charges fix only the geometric deformations of the CY space including the **NS-NS** B-field. This involves the complex structure deformations, the complexified Kähler deformations or both.

$$p = 0 \quad \text{black hole charges fix} \rightarrow \text{geometric moduli}$$

- The dilaton could be fixed by the dyonic object.

$$p = 3 - n \quad \text{Dyonic black brane charges fix} \rightarrow \text{the dilaton}$$

- The moduli coming from **R-R** gauge fields on CY cycles should be fixed by the higher dimensional black objects charges, like strings and branes.

$$p \neq 0, 3 - n \quad \text{black brane charges fix} \rightarrow \text{R-R stringy moduli.}$$

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