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Heat Transfer In A Two-Layered Blood Flow Model In A Narrow Tube In Presence Of Magnetic Field

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abstract

Blood flow in a narrow tube is described using the two-layered model in presence of an external uniform magnetic field. The model consists of a core region, enriched with various types of blood cells and a cell-free peripheral plasma layer. Also the constant heat flux convective heat transfer to fully developed blood flow is studied. The velocity and temperature profiles are determined. The expressions for friction-factor-Reynolds number product and Nusselt number are found and their natures are shown graphically.
Keywords:two-layered-model, convective heat transfer, friction-factor-Reynolds number, Nusselt number.

I. INTRODUCTION

The regulation of body temperature is mainly done by the circulatory system. Body heat produced by the skeletal muscle is in general removed by the convection heat transfer of blood [1]. It is also observed that the smaller blood vessels are more actively engaged in the heat exchange with the surroundings. So, the thermally important blood vessels are of very small diameter and are in the range of 50 500 μm [2,3]. Researchers have made number of investigations regarding the heat transfer between the small blood vessels and their surrounding tissues [4,5,6]. Wang [7] studied the fully developed heat transfer in a narrow vessel using the two-fluid model. Convective heat transfer of blood is of much importance in the use of various clinical aspects such as local hyperthermia, cryrosurgery for tumor treatment etc.

To determine the convective heat transfer from a blood vessel, we have to first find out the velocity profile. In larger vessels the nature of blood is almost homogeneous and we may readily compute the heat transfer using parabolic (i.e., Poiseuille) velocity distribution [8]. But for flow in narrow vessels, the blood cells tend to migrate along the centre of the vessel which give rise to a core region in which the concentration of blood cells is high, and a slower moving peripheral plasma region which is almost cell free [9,10]. So the flow of blood in a narrow vessel may be explained by a two-layered model [11,12,13]. A good number of models have been developed regarding this phenomenon [14,15]. Pries et al. derived empirical relationships of the relative apparent viscosity and mean tube hematocrit from in vitro [16,17] and in vivo [18].

In the present analysis we study the effect of an external magnetic field on the convective heat transfer of blood in a narrow vessel. The analytical expressions of velocity for blood and temperature in core region as well as in peripheral plasma region are determined. Also the expressions for friction-factor-Reynolds number and Nusselt number are obtained. The effect of magnetic field and thickness of core region on them are shown through graphs.

II. MATHEMATICAL FORMULATIONS AND SOLUTIONS

We consider a two-fluid model for blood flow within a narrow cylindrical tube of radius R and length L. The core region is assumed to be enriched with various kinds of blood cells with radius r_c and effective viscosity μ_c. The cell free plasma layer has the effective viscosity μ_0. Thus this model consists of two immiscible fluids (Fig. 1).

Let us consider r and z be the radial and axial directions in the tube, u_c and u_0 be the velocity of blood in the core region and peripheral plasma region respectively along the axial direction.

The basic equations may be written in the form

$$-\frac{\partial p}{\partial z}+\frac{\mu_c}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_c}{\partial r}\right)-\sigma B_0^2 u_c=0 \quad ; \quad 0\leq r\leq r_c \tag{2.1}$$

for the core region and

$$-\frac{\partial p}{\partial z}+\frac{\mu_0}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_0}{\partial r}\right)=0 \quad ; \quad r_c< r\leq R \tag{2.2}$$

for peripheral plasma region, where $\frac{\partial p}{\partial z}$ is the pressure gradient, σ is the electrical conductivity of the medium and B_0 is the transverse component of the magnetic field. As the peripheral plasma layer is considered cell free, the effect of B_0 is negligible in this region.

The boundary conditions for velocity are given as

(a) u_c is bounded on the axis and because of symmetry, the velocity gradient vanishes along the axis of the tube:

$$\frac{\partial u_c}{\partial r}=0 \qquad \text{at } \ \mathrm{r}=0. \tag{2.3}$$

(b) There is no slip on the wall:

$$u_0=0 \qquad \text{at } \ \mathrm{r}=\mathrm{R}. \tag{2.4}$$

(c) The velocity and shear stress are continuous at the interface of the two layers:

$$(i) \qquad u_c|_{r=r_c}=u_0|_{r=r_c}\,, \tag{2.5}$$

$$(ii) \qquad \mu_c\left.\frac{\partial u_c}{\partial r}\right|_{r=r_c}=\mu_0\left.\frac{\partial u_0}{\partial r}\right|_{r=r_c}. \tag{2.6}$$

Taking $\xi=\frac{r}{R}$, $\lambda=\frac{r_c}{R}$, $P=-\frac{1}{R}\frac{\partial p}{\partial z}$ and using the boundary conditions (2.3)-(2.6) we obtain the solutions of (2.1) and (2.2) as

$$\left.\begin{array}{l} u_c=\frac{J_0(a\xi)}{J_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right)-\frac{P}{\sigma B_0^2}\right\}+\frac{P}{\sigma B_0^2};0\leq\xi\leq\lambda \\ u_0=\frac{PR^2}{4\mu_0}\left(1-\xi^2\right);\lambda\leq\xi\leq 1 \end{array}\right\} \tag{2.7}$$

where $a=iB_0R\sqrt{\frac{\sigma}{\mu_c}}$.

Let V be the mean velocity, so that πV gives the total flow. Therefore

$$V=2\left\{\int_0^{r_c}u_c r dr+\int_{r_c}^{R}u_0 r dr\right\}$$

$$=\frac{2R^2\lambda}{\alpha}\frac{J_1\left(a\lambda\right)}{J_0\left(a\lambda\right)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right)-\frac{P}{\sigma B_0^2}\right\}+\frac{PR^2\lambda^2}{\sigma B_0^2}+\frac{PR^2}{8\mu_0}\left(1-\lambda^2\right)^2. \tag{2.8}$$

Now we consider T(r,z) as the temperature and q as the constant heat flux applied on the wall. Assuming T_c as the temperature on the core region and T_0 as the temperature on peripheral plasma layer we may construct the forced convection equations as

$$P_c u_c \frac{\partial T_c}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_c}{\partial r}\right), \tag{2.9}$$

for the core region and

$$\beta P_c u_0 \frac{\partial T_0}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_0}{\partial r}\right), \tag{2.10}$$

for the peripheral plasma region, where P_c is the Peclet number and $\beta\left(=\frac{\kappa_c}{\kappa_0}\right)$ is the ratio of the thermal diffusivities.

Let us introduce two new parameters; $\gamma\left(=\frac{k_c}{k_0}\right)$, the ratio of conductivities and T_m, the mean temperature given by

$$T_m(z) = \frac{2}{V}\left\{\int_0^{r_c} u_c T_c r dr + \int_{r_c}^{R} u_0 T_0 r dr\right\}. \tag{2.11}$$

For constant flow problem, all the temperatures must have same constant gradient (say A) in the axial direction. Thus we may write

$$\frac{\partial T_c}{\partial z} = \frac{\partial T_0}{\partial z} = \frac{\partial T_m}{\partial z} = A. \tag{2.12}$$

The boundary conditions for temperature are given as :
(a) T_c is bounded on the axis,
(b) the heat flux on the wall is

$$\frac{\partial T_0(R,z)}{\partial r} = \gamma, \tag{2.13}$$

(c) the temperature and fluxes match on the interface, i.e.,

$$(i) \quad T_c(r_c,z) = T_0(r_c,z), \tag{2.14}$$

$$(ii) \quad \gamma\frac{\partial T_c(r_c,z)}{\partial r} = \frac{\partial T_0(r_c,z)}{\partial r}, \tag{2.15}$$

(d)

$$T_m(0) = 0. \tag{2.16}$$

Applying the aforementioned boundary conditions and the same non-dimensionalisation scheme used for velocity profile, we obtain the solutions of (2.9) and (2.10) as

$$T_c = Az - \frac{P_c R^2 A J_0(a\xi)}{a^2 J_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right) - \frac{P}{\sigma B_0^2}\right\} + \frac{PP_cR^2A}{4\sigma B_0^2}\xi^2 + C_1, \tag{2.17}$$

$$T_0 = Az + \frac{\beta PP_cR^4A}{4\mu_0}\left\{\frac{\xi^2}{4} - \frac{\xi^4}{16} + C_2\ln\xi + C_3\right\}. \tag{2.18}$$

The constants A, C_1, C_2 and C_3 can be determined from the boundary conditions (2.13)-(2.16) and are given in the appendix.

The friction-factor-Reynolds number product fRe is defined as [7]
fRe $=\frac{8}{V}$

$$= \frac{8}{\frac{2R^2\lambda}{\alpha}\frac{J_1(a\lambda)}{J_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right)-\frac{P}{\sigma B_0^2}\right\}+\frac{PR^2\lambda^2}{\sigma B_0^2}+\frac{PR^4}{8\mu_0}\left(1-\lambda^2\right)^2}. \tag{2.19}$$

We also obtain the gauge for the heat transfer, known as Nusselt number, and its value is computed numerically using the formula

$$Nu = \frac{2}{T_0\left(R,z\right)-T_m\left(z\right)}. \tag{2.20}$$

Numerically it can be shown that all the above analytical results agree with the previous results [13,7] when $B_0 \to 0$.

III. NUMERICAL RESULTS AND DISCUSSIONS

The velocity in the core region and in the peripheral plasma layer are obtained in equation (2.7). The values of these velocity profiles are computed and depicted through the figs. 2 and 3. The effect of the transverse component of the magnetic field B_0 is also shown in that figures. In those figures we observe that the velocity decreases with the increase in the value of B_0.

In fig. 4, the nature of velocity profile is shown for different values of λ taking B_0=0.02. Here we observe that the velocity profile decreases with the increase in the values of λ.

In fig. 5, the behavior of temperature profile is shown for λ= 0.7. From this figure it is clear that the transfer of heat occurs mainly in the peripheral plasma layer. In the core region as the value of r increases the value of temperature profile increases negligibly; but in the peripheral plasma layer the value of temperature profile increases much more rapidly with the increase in the values of r. Also the heat transfer decreases as the magnetic field B_0 increases.

From equation (2.19) the values of friction-factor-Reynolds number fRe are computed and the effect of the magnetic field B_0 on fRe is shown through the fig. 6. Here we observe that the value of fRe increases as the value of B_0 increases.

The values of Nusselt number Nu are calculated using equation (2.20) and the variation of Nu against λ is shown on fig. 7. Here we observe that the Nusselt number is effected largely by the magnetic field B_0. It increases with the increase in the values of B_0. For λ=0, Nu becomes 0.436 and whenever λ tends to one, Nu tends to zero for any value of B_0.

IV. CONCLUSIONS

In biological systems, the regulation of body temperature is of great importance and is mainly maintained by the blood vessels. From our present discussions the amount and nature of heat transfer through the blood vessels can be calculated. Using an external magnetic field we can regulate the heat transfer.

V. APPENDIX

From the boundary condition (2.13) we obtain the value of A as

$$A = \frac{R\gamma}{\frac{P_cR^2\lambda J_1(a\lambda)}{aJ_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right)-\frac{P}{\sigma B_0^2}\right\}+\frac{PP_cR^2\gamma\lambda^2}{2\sigma B_0^2}-\frac{\beta PP_cR^4}{4\mu_0}\left(\frac{\lambda^2}{2}-\frac{\lambda^4}{4}\right)+\frac{\beta PP_cR^4}{16\mu_0}}$$

and from the boundary condition (2.15) we obtain the value of C_2 as

$$C_2 = \frac{4\mu_0}{\beta R^2}\left[\frac{\gamma\lambda J_1\left(a\lambda\right)}{aJ_0\left(a\lambda\right)}\left\{\frac{R^2}{4\mu_0}\left(1-\lambda^2\right)-\frac{1}{\sigma B_0^2}\right\}+\frac{\gamma\lambda^2}{2\sigma B_0^2}-\frac{\beta R^2}{4\mu_0}\left(\frac{\lambda^2}{2}-\frac{\lambda^4}{4}\right)\right].$$

From the boundary conditions (2.14) and (2.16) we obtain two relations connecting C_1 and C_3 namely

$$C_1 = PP_cR^2A\left[\frac{\beta R^2}{4\mu_0}C_3 + \left\{\frac{\beta R^2}{4\mu_0}\left(\frac{\lambda^2}{4} - \frac{\lambda^4}{16} + C_2\ln\lambda\right) + \frac{R^2}{4\mu_0 a^2}\left(1-\lambda^2\right) - \frac{1}{\sigma B_0^2 a^2} - \frac{\lambda^2}{4\sigma B_0^2}\right\}\right]$$

and

$$\frac{2R^2}{V}\left[\frac{Az\lambda J_1(a\lambda)}{aJ_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right) - \frac{P}{\sigma B_0^2}\right\} + \frac{PAz\lambda^2}{2\sigma B_0^2} - \frac{P_cR^2A\lambda^2}{2a^2J_0^2(a\lambda)}\left\{J_0^2(a\lambda) + J_1^2(a\lambda)\right\} - \frac{PP_cR^2A\lambda J_1(a\lambda)}{\sigma B_0^2 a^3 J_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right) - \frac{P}{\sigma B_0^2}\right\} + \frac{PP_cR^2A}{4\sigma B_0^2 J_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right) - \frac{P}{\sigma B_0^2}\right\}\int_0^\lambda \xi^2 J_0(a\xi)\,d\xi + \frac{P^2P_cR^2A\lambda^3}{12\sigma^2B_0^4} + \frac{C_1\lambda J_1(a\lambda)}{aJ_0(a\lambda)}\left\{\frac{PR^2}{4\mu_0}\left(1-\lambda^2\right) - \frac{P}{\sigma B_0^2}\right\} + \frac{PC_1\lambda^2}{2\sigma B_0^2}\right] + \frac{2R^2}{V}\left[\frac{PR^2Az}{4\mu_0}\left(\frac{1}{4} - \frac{\lambda^2}{2} + \frac{\lambda^4}{4}\right) + \frac{\beta P^2P_cR^6A}{16\mu_0^2}\left\{\frac{C_3}{2}\left(1-\lambda^2\right) + \left(\frac{1}{16} - \frac{C_3}{4}\right)\left(1 - \frac{\lambda^4}{4}\right) - \frac{5}{96}\left(1-\lambda^6\right) + \frac{1}{128}\left(1-\lambda^8\right) + C_2\left(-\frac{1}{4} - \frac{\lambda^2}{2}\ln\lambda + \frac{\lambda^2}{4}\right) - C_2\left(-\frac{1}{16} - \frac{\lambda^4}{4}\ln\lambda + \frac{\lambda^4}{16}\right)\right\}\right] = 0.$$

Solving these two equations numerically we get the values for C_1 and C_3.

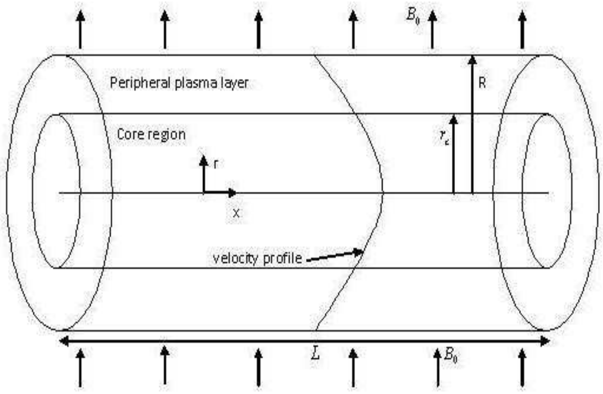

Fig. 1 : Schematic diagram of the model

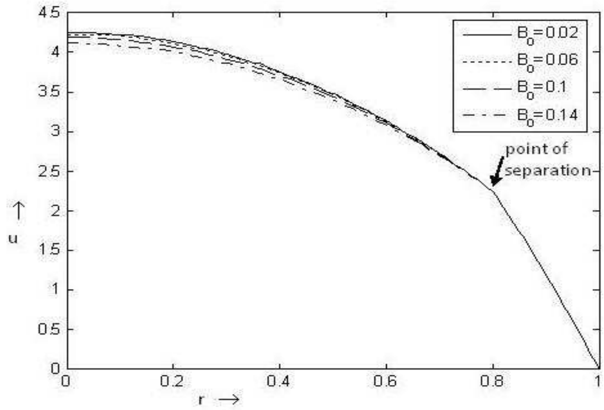

Fig. 2 : Velocity profile taking λ =0.8 .

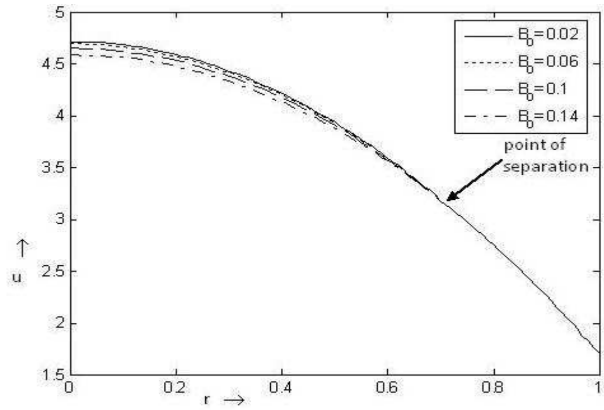

Fig. 3 : Velocity profile taking λ =0.7 .

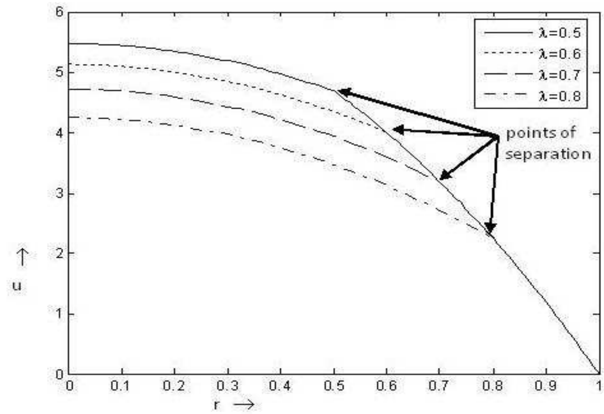

Fig. 4 : Velocity profile taking B_o=0. 02.

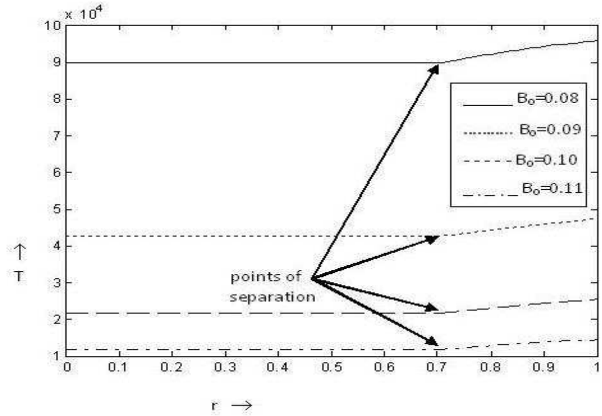

Fig. 5 : Temperature profile taking λ =0. 7.

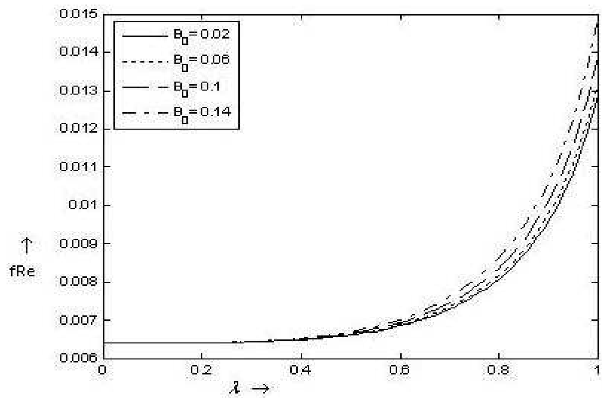

Fig. 6 : f Re as a function of λ .

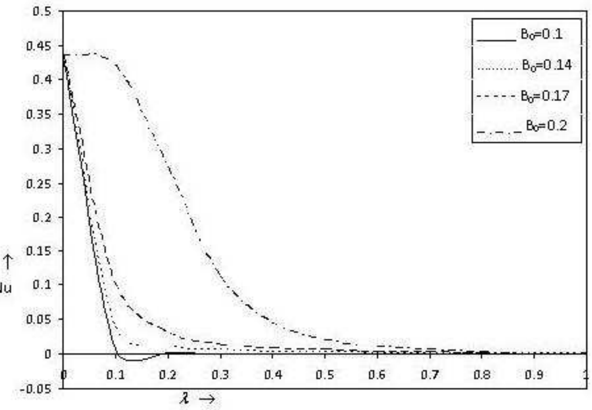

Fig. 7 : Nusselt number as a function of λ .

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