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Superstring Theory on AdS Spaces in The Penrose Limit

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Abstract

We give a brief review on AdS/CFT correspondence then we descuss how string spectrum in flat space and plane waves arises from the large N limit of $U(N)$ $\mathcal{N}=4$ super Yang Mills. We also explain the operator-state correspondence for ths case of $AdS_5 \times S^5$ and $AdS_5 \times S^5/\mathbf{Z}_k$ in some details.

I. INTRODUCTION

AdS/CFT correspondence[1]-[4] has lead to deep understandings of string theory and field theory. However, until recently, much of the progress in this direction has been limited to supergravity approximations due to the difficulty when one has Ramond-Ramond background*. Recently, it has been shown that string theory can be fully solved in the pp wave background even in the presence of RR flux[5] in the light cone Green Schwarz formalism. Shortly after this development Berenstein *et al*[6] have put forward an exciting proposal that tests AdS/CFT correspondence beyond the supergravity approximation. More specifically, they have related closed string states in pp wave background with operators of the dual $\mathcal{N}=\triangle$ SYM with large R charge $J \sim \sqrt{N}$ and finite $\Delta - J$. Many interesting papers have subsequently followed[7]-[23].

In these notes, we give an overview of the AdS/CFT correspondence and describe how we can reproduce the spectrum of strings on $AdS_5 \times W$ from the gauge theory point of view, where $W = S^5$ or $S^5/\mathbf{Z_k}$.

These notes are organized as follows: in the section 2 we review the main lines of the AdS/CFT correspondence. In section 3, we descuss the plane waves and strings on their backgrounds. In section 4, we show how to get the strings on plane waves from AdS spaces. In section 5, we analyse the operator-state correspondence for $AdS_5 \times S^5$ and its orbifold $AdS_5 \times S^5/\mathbf{Z_k}$. The last section is devoted to the conclusion.

II. ADS/CFT CORRESPONDENCE: A SHORT REVIEW

During these last few years, one was able to describe large N gauge theories from string theory. Here we give a brief review about the Maldacena conjecture saying that the $\mathcal{N}=\triangle$ $U(N)$ super Yang-Mills theory in four dimension is the same as (or dual to) the type IIB superstring theory on $AdS_5 \times S^5$.

Euclidean version of the AdS geometry

Consider an Euclidean space $\mathbf{R^{d+1}}$ parameterized by the set f coordinates $y_0, y_1, ..., y_d$, and let $\mathbf{B}_{d+1}$ denotes the open unit ball $\sum_{i=0}^{d} y_i^2 < 1$. The Anti-de Sitter space AdS_{d+1} can be identified with $\mathbf{B}_{d+1}$ with the metric

$$ds^2 = 4\frac{\sum_{i=0}^{d} y_i^2}{(1-|y|^2)^2} \tag{2.1}$$

From the open unit ball we can define $\bar{B}_{d+1}$ the closed unit one with $\sum_{i=0}^{d} y_i^2 \leq 1$ such that its

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boundary is the sphere $\sum_{i=0}^{d} y_i^2 = 1$. We see that the metric (metric) is singular at $|y|^2 = 1$. To resolve this problem we pick a function f on $\mathbf{B}_{d+1}$ wich is positive on $\overline{\mathbf{B}}_{d+1}$ for instance $f = 1 - |y|^2$. This function is defined up to a conformal transformation $f \to fe^{\omega}$ for any arbitrary ω and under this transformation the metric transforms as

$$d\tilde{s}^2 \to e^{2\omega} ds^2 \tag{2.2}$$

Finally, the AdS_{d+1} can be viewed as the upper half plane $x^0 > 0$ in a space of coordinates $x^0, x^1, ..., x^d$ and metric

$$ds^2 = \frac{1}{x^2} \sum_{i=0}^{d} \left(dx_i^2\right). \tag{2.3}$$

In this representation the boundary is a copy of $\mathbf{R^d}$ at $x^0 = 0$ plus a point P at in infinity.

Conformal algebra

The conformal group is defined as the transformation group preserving the form of the metric up to a scale factor

$$g_{\mu\nu}(x) \longrightarrow \Omega^2(x)\, g_{\mu\nu}(x) \tag{2.4}$$

It is generated, in Minkowski space, by Poincaré, scale and special conformal transformations. The conformal algebra can be constructed by the following commutation relations

$$\begin{aligned} [M_{\mu\nu}, P_\rho] &= -i\left(\eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu\right); \\ [M_{\mu\nu}, K_\rho] &= -i\left(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu\right); \\ [M_{\mu\nu}, M_{\rho\sigma}] &= -i\eta_{\mu\rho} M_{\nu\sigma} + \text{permutations}; \\ [P_\mu, K_\nu] &= 2iM_{\mu\nu} - 2i\eta_{\mu\rho} D, \\ [D, K_\mu] &= iK_\mu, \\ [P_\mu, D] &= iP_\mu. \end{aligned} \tag{2.5}$$

with the all other relations vanish. $M_{\mu\nu}$, P_ρ, D and K_μ been the Lorentz transformation, translation scale transformation and special conformal transformation generators respectively.

Superconformal algebra

Existing only for in dimension $d \le 6$, superconformal algebra is generated in addition to the above commutation relations by the following extra ones

$$\begin{aligned} [D, Q] &= -\frac{i}{2} Q; & \{Q, Q\} &\simeq P, \\ [D, S] &= \frac{i}{2} S; & \{S, S\} &\simeq K, \\ [K, Q] &\simeq S; & \{Q, S\} &\simeq M + D + R, \\ [P, S] &\simeq Q; \end{aligned} \tag{2.6}$$

where S and R are the supersymmetry and R-symmetry generators.

The Conjecture

The complete effective action of non massives modes contains tree terms

$$\mathcal{S} = \mathcal{S}_{\text{vol}} + \mathcal{S}_{\text{brane}} + \mathcal{S}_{\text{int}} \tag{2.7}$$

where

- The volume action is a free quadratic part describing the propagation of free non massive modes of the supergravity including graviton and given by

$$\mathcal{S}_{\text{vol}} \sim \frac{1}{2\kappa^2} \int \sqrt{g}\mathcal{R} \sim \int (\partial h)^2 + \kappa\, (\partial h)^2\, h + ... \tag{2.8}$$

  with $g = \eta + \kappa h$.

- The brane $\mathcal{S}_{\text{brane}}$ action is defined on its (3+1)-dimensional world-volume, it contains $\mathcal{N} = \triangle$ super Yang Mills lagrangian plus higher derivative terms

$$\mathcal{S}_{\text{brane}} \sim \frac{1}{g} \int F^2 + \alpha' F^4. \tag{2.9}$$

  Finally $\mathcal{S}_{\text{int}}$ describe the interactions between the brane modes and those of the volume

$$\mathcal{S}_{\text{int}} \sim \frac{1}{g} \int hF^2. \tag{2.10}$$

  The totale action can be written as

$$\mathcal{S} = \int \mathrm{d}^{10}x \left[(\partial h)^2 + \kappa\, (\partial h)^2\, h + ...\right] \frac{1}{g} \int F^2 + \alpha' F^4 + \frac{1}{g} \int \kappa \tilde{h} F^2 \tag{2.11}$$

  with $h = \kappa\tilde{h}$.

In the low energy limit $l_s \to 0$ ($\alpha' \to 0$), the interaction lagrangian vanishes and also do all higher derivative terms in the brane action. We are left with just the pure gauge theory $\mathcal{N} = \triangle\ U(N)$ in $3+1$ dimension, known to be a conformal field theory, and the supergravity in the bulk becomes free.

In Conclusion, we have two decoupled systems in this limit of low energy: In one hand a free gravity in the bulk and in the other hand 4-dimensional gauge theory

$$\mathcal{S} = \int \mathrm{d}^{10}x\, (\partial h)^2 + \frac{1}{g} \int F^2. \tag{2.12}$$

Solution of the supergravity

Now let us consider the same system but from a different view. D-branes are charged massive objects behaving as sources for different fields of the supergravity. The type IIB SUGRA contains

$$\mathcal{S}=\int\sqrt{g}R+(\nabla\phi)^2+F_5^2+\text{other termes} \quad (2.13)$$

we can find a D3 brane solution of supergravity of the form

$$\begin{aligned} ds^2 &= f^{-1/2}\left(-dt^2+dx_1^2+dx_2^2+dx_3^2\right) \\ &\quad +f^{1/2}\left(dr^2+r^2d\Omega_5^2\right) \\ F_5 &= (1+*)\,dtdx_1dx_2dx_3df^{-1/2}, \\ f &= 1+\frac{R^4}{r^4}; \quad R^4\equiv 4\pi g_s\alpha'^2N. \end{aligned} \quad (2.14)$$

The theory at low energy contains two decoupled pieces: One is a free supergravity in the bulk and the second is the geometry of the near horizon region. In this near horizon region, $r\ll R$, with the approximation $f\sim R^4/r^4$, and the geometry becomes

$$\begin{aligned} ds^2 &= \frac{r^2}{R^2}\left(-dt^2+dx_1^2+dx_2^2+dx_3^2\right) \\ &\quad +R^2\frac{dr^2}{r^2}+R^2d\Omega_5^2 \end{aligned} \quad (2.15)$$

which is the geometry of $AdS_5\times S^5$.

Conclusion

We see, from two points of view: From the field theory of open strings living on the brane and from the description of the supergravity, that we have two decoupled theories in the low energy limit. In both cases one of the decoupled systems is the supergravity in flat space.

Field $\longleftrightarrow$ operator

To work out more explicitly this correspondence we have to use the following formula

$$\begin{aligned} \langle e^{\int d^4x\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{\text{CFT}} &= \mathcal{Z}_{\text{string}}[\phi(\vec{x},\ddagger)\mid_{\ddagger=\prime} \\ &= \phi_0(\vec{x})] \end{aligned} \quad (2.16)$$

with $\mathcal{O}$ is a conformal field theory operator and where

- The left hand side of the equation is the generating functional of correlation functions in field theory in taking the functional derivatives by respect to ϕ_0 and than set $\phi_0=0$.
- The right hand side of the equation is the complete partition function of the string theory with some boundary condition on ϕ: It goes to ϕ_0 when one approaches the boundary of AdS.

Any propagating field in the AdS space is in one-to-one correspondence with an operator in conformal theory. There is so a relation between the mass m of the field ϕ and the scale dimension Δ of the operator in the conformal field theory. By computing the correlation functions we van show the following relations

Scalars:	$\Delta_\pm=\frac{1}{2}\left(d\pm\sqrt{d^2+4m^2}\right)$	(2.17)
Spinors:	$\Delta=\frac{1}{2}(d+2\lvert m\rvert)$	
Vectors:	$\Delta_\pm=\frac{1}{2}\left(d\pm\sqrt{(d-2)^2+4m^2}\right)$	
$p-$forms:	$\Delta_\pm=\frac{1}{2}\left(d\pm\sqrt{(d-2p)^2+4m^2}\right)$	
Spin-3/2 :	$\Delta=\frac{1}{2}(d+2\lvert m\rvert)$	

III. PLANE WAVES AND STRINGS ON THEIR BACKGROUNDS

A plane wave is a geometry of the general form

$$ds^2=-4dx^+dx^-+H\left(x^+,y\right)\left(dx^+\right)^2+dy^idy^i \quad (3.1)$$

where H is a function independent of x^- and can be chosen quadratic in y^i,

$$H=-A_{ij}\left(x^+\right)y^iy^j. \quad (3.2)$$

For a matrix A independent of x^+, the metric (3.1) becomes

$$ds^2=-4dx^+dx^--A_{ij}y^iy^j\left(dx^+\right)^2+dy^idy^i. \quad (3.3)$$

Now we want to consider string theory on this background. In fact, if we choose the light cone gauge $x^+=\tau$ with τ the worldsheet time, the bosonic action reduces to

$$\begin{aligned} S=\frac{1}{2\pi\alpha'^2}\int dt\int_0^{\pi\alpha'|p_-|}d\sigma\frac{1}{2}\Big[\dot{y}^2 \\ -y'^2-A_{ij}(\tau)\,y^iy^j\Big] \end{aligned}$$

with the momenta $p_\pm$ generating translations in the $x^\pm$ directions. If A is x^+ independent we do not have particle creation on the world sheet. We will consider only the case of type IIB where $A_{ij}=\mu\delta_{ij}$ and F_5 is given by

$$\begin{aligned} F &= dx^+\varphi, \\ \varphi &= \mu\left(dy^1dy^2dy^3dy^4+dy^5dy^6dy^7dy^8\right); \end{aligned} \quad (3.4)$$

so that we get the metric

$$ds^2=-4dx^+dx^--\mu^2y^iy^j\left(dx^+\right)^2+dy^idy^i. \quad (3.5)$$

In the superstring case we can start with the Green Schwarz action and choose light cone gauge $x^+=\tau$ and $\Gamma_-\theta^\alpha,\ \alpha=1,2$. Then we obtain the action

$$\begin{aligned} S=\frac{1}{2\pi\alpha'^2}\int dt\int_0^{\pi\alpha'|p_-|}d\sigma \\ \left[\frac{1}{2}\dot{z}^2-\frac{1}{2}z'^2-\frac{1}{2}\mu^2z^2+i\overline{S}\left(\partial/+\mu I\right)S\right] \end{aligned} \quad (3.6)$$

where $I = \Gamma^{1234}$ and S is a Majorana spinor on the wordsheet and a positive chirality $SO(8)$ spinor under rotations in the eight transverse directions. We can quantize this action by expanding all fields in Fourier modes on the circle labeled by σ giving rise to a harmonic oscillator for each mode. The light cone Hamiltonian is

$$2p^- = -p_+ = H_{lc} = \sum_{n=-\infty}^{+\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha'|p_-|/2)}} \tag{3.7}$$

N_n denotes the total occupation number including bosons and fermions and n labels the Fourier modes. Note that the ground state energy of bosonic oscillators is canceled by that of the fermionic oscillators.

The constraint on the momentum in the σ direction reads

$$P = \sum_{n=-\infty}^{+\infty} nN_n = 0. \tag{3.8}$$

Two interesting limits to be considered:

- In the limit that μ is very small

$$\mu\alpha'|p_-| \ll 1? \tag{3.9}$$

  we recover the flat space spectrum with a flat space metric.
- the opposite limit

$$\mu\alpha' p^+ \gg 1? \tag{3.10}$$

  corresponds to a highly curved background with RR fields. In this limit all the low lying string oscillator modes have almost the same energy.

IV. STRING THEORY ON PLANE WAVE FROM ADS

$AdS_5 \times S^5$ case

Let us consider the trajectory of a particle that is moving very fast along the S^5 and focus on the geometry that this particle sees. The $AdS_5 \times S^5$ metric can be written as

$$ds^2 = R^2\left[-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2\right]. \quad +d\psi^2\cos^2\theta + d\theta^2 + \sin^2\theta d\Omega_3'^2\Big] \tag{4.1}$$

We want to consider a particle moving along the ψ direction and sitting at $\rho = 0$ and $\theta = 0$. For this we introduce coordinates $\widetilde{x}^{\pm} = \frac{t\pm\psi}{2}$ and then performing the rescaling

$$x^+ = \widetilde{x}^+, \quad x^- = R^2\widetilde{x}^-, \quad \rho = \frac{r}{R}, \quad \theta = \frac{y}{R}, \quad R \to \infty \tag{4.2}$$

In this limit the metric (4.1) becomes

$$ds^2 = -4dx^+dx^- - \left(\overrightarrow{r}^2 + \overrightarrow{y}^2\right)\left(dx^+\right)^2 + d\overrightarrow{r}^2 + d\overrightarrow{y}^2 \tag{4.3}$$

where $\overrightarrow{y}$ and $\overrightarrow{r}$ parameterize points on R^4. The mass parameter μ can be introduced by the rescaling $x^- \to x^-/\mu$ and $x^+ \to \mu x^+$.

Now let us see how the energy and angular momentum along scale in the limit (4.2).

$$\begin{array}{ccc} AdS & & CFT \\ E = i\partial_t J = -i\partial_\psi & \Longleftrightarrow & \text{Energy}R - \text{chargeof a state on } S \times R \end{array} \tag{4.4}$$

We can also say that $E = \Delta$ is the conformal dimension of an operator on R^4. We find that

$$\begin{aligned} 2p^- &= -p_+ = i\partial_{x^+} = i\partial_{\widetilde{x}^+} \\ &= i\left(\partial_t + \partial_\psi\right) = \Delta - J \\ 2p^+ &= -p_- = -\frac{\widetilde{p}_-}{R^2} = \frac{1}{R^2} i\partial_{\widetilde{x}^-} \\ &= \frac{1}{R^2} i\left(\partial_t - \partial_\psi\right) = \frac{\Delta + J}{R^2} \end{aligned} \tag{4.5}$$

Configurations with fixed non zero p_- in the limit (4.2) correspond to states in AdS with large angular momentum $J \sim R^2 \sim (gN)^{1/2}$. The contribution of each oscillator to $\Delta - J$ in terms of the Yang-Mills parameters is

$$(\Delta - J)_n = \omega_n = \sqrt{1 + \frac{4\pi g N n^2}{J^2}} \tag{4.6}$$

The limit is performed in two ways:

- We take the $N \to \infty$ limit keeping the string coupling g fixed and we focus on operators with $J \sim N^{1/2}$ and $\Delta - J$ fixed.
- We could first take the 't Hooft limit $g \to 0$, gN =fixed, and then after taking this limit, we take the limit of large 't Hooft coupling keeping $J/\sqrt{gN}$ fixed and $\Delta - J$ fixed. This gives us a plane wave background with zero string coupling.

Since we are interessted in the free string spectrum of the theory it will be more convenient to take this second limit

$AdS_5 \times S^5/\mathbf{Z_k}$ case

Now let us consider the metric (4.3), by taking the $\mathbf{Z_k}$ orbifold of the R^4 subspace parameterized by $\overrightarrow{y}$ we can reduce the 32 supersymmetries to 16 ones.This orbifold action is defined by

$$g: \quad \left(z^1, z^2\right) \to \left(\omega z^1, \omega z^2\right) \tag{4.7}$$

where $\omega = e^{\frac{2\pi i}{k}}$ and $z^1 \equiv \frac{1}{\sqrt{2}}\left(y^6 + iy^7\right)$, $z^2 \equiv \frac{1}{\sqrt{2}}\left(y^8 - iy^9\right)$. While on the spacetime fields it acts as $g = \exp\left[\frac{2\pi i}{k}\left(J_{67} - J_{89}\right)\right]$, J_{67} and J_{89} being the rotation generators in the 67 and 89 planes, respectively.

In this case the constraint (3.8) becomes

$$\sum_{n\in\mathbf{Z}} nN_n = 0 \tag{4.8}$$

$$\sum_{n\in\mathbf{Z}} n(q)\left(N_{n(q)} - \overline{N}_{-n(q)}\right) = \tag{4.9}$$

$$\sum_{n\in\mathbf{Z}} \left(n(q) N_{n(q)} + n(-q)\overline{N}_{n(-q)}\right) = 0 \tag{4.10}$$

with $q = 0, \ldots, k-1$ label the twisted sector and $n(q) = n + \frac{q}{k}$ $(n \in \mathbf{Z})$.

The light cone Hamiltonian in the q-th twisted sector is given by

$$H_{lc}(q) = \sum_{n\in\mathbf{Z}} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}} + \sum_{n\in\mathbf{Z}} \left(N_{n(q)} + \overline{N}_{-n(q)}\right)\sqrt{\mu^2 + \frac{n(q)^2}{(\alpha' p^+)^2}} \tag{4.11}$$

The first sum is over those oscillators which are not sensitive to the orbifold. Positive modes label left movers, negative ones right movers, N_n ($N_{n(q)}$ and $\overline{N}_{-n(q)}$) is the total occupation number of bosons and fermions.

For $\Delta - J \ll J$ The light cone Hamiltonian (**??**) implies that on the gauge theory side there are operators obeying the following relation between the dimension Δ and the $U(1)_R$ charege J (we set $\mu \equiv 1$)

$$\begin{aligned}(\Delta - J)_n &= \sqrt{1 + g_{eff}n^2} \\ (\Delta - J)_{n(q)} &= \sqrt{1 + g_{eff}n(q)^2}.\end{aligned} \tag{4.12}$$

V. STRING $\leftrightarrow \mathcal{N} = \triangle$ SUPER YANG MILLS

$AdS_5 \times S^5$ case

The goal is the following:Consider states carrying large R charge $J \sim \sqrt{N}$ (J being the $SO(2)$ generator rotating two of the six scalar fields) and then look for spectrum of states with finite $\Delta - J$ in this limit.

- The $\Delta - J = 0$ case

  There is a unique single trace operator with this choice: $Tr[Z^J]$ $(Z = \phi^5 + i\phi^6)$. It is associated to the vacuum state in light cone gauge, which is the unique state with zero light cone Hamiltonian. We have the correspondence

$$\frac{1}{\sqrt{J}N^{J/2}} Tr[Z^J] \leftrightarrow |0, p_+\rangle_{l.c.} \tag{5.1}$$

- $\Delta - J = 1$

  Fields existing in the theory:

8 bosonic modes	*8 fermionic modes*
$\phi^i, \quad i = 1, \ldots, 4$	$\chi^a_{J=1/2}$,
$D_i Z = \partial_i Z + [A_i, Z]$	$a = 1, \ldots, 8$

(5.2)

By acting by one of the rotations of S^5 which do not commute with the $SO(2)$ symmetry we create states of the form

$$\frac{1}{\sqrt{J}} \sum_l \frac{1}{\sqrt{J} N^{J/2+1/2}} Tr[Z^l \phi^r Z^{J-l}] = \frac{1}{N^{J/2+1/2}} Tr[\phi^r Z^J] \tag{5.3}$$

with ϕ^r $r = 1, \ldots, 4$ one of the scalars neutral under J.

And we have the following correspondence with a state with two excitations

$$\frac{1}{\sqrt{J}} \frac{1}{N^{J/2+1}} \sum_{l=1}^{J} Tr[\phi^r Z^l \psi^b_{J=1/2} Z^{J-l}] \leftrightarrow a_0^{+k} S_0^{+b} |0, p_+\rangle_{l.c.} \tag{5.4}$$

a^+ and S^+ being the the bosonic and fermionic creation operators respectively.

- Non supergravity modes

  In this case the operator should include a phase for instance the correspondence for two insertions is

$$\frac{1}{\sqrt{J}} \sum_{l=1}^{J} \frac{1}{N^{J/2+1}} Tr[\phi^3 Z^l \phi^4 Z^{J-l}] e^{\frac{2\pi n l}{J}} \leftrightarrow a_n^{+8} a_{-n}^{+7} |0, p_+\rangle_{l.c.} \tag{5.5}$$

- Rule

  Each string oscillator corresponds to the insertion of a $\Delta - J = 1$ field, summing over all positions with an n dependent phase, according to

$$\begin{aligned} a^{+i} &\to D_i Z, && \text{for } i = 1, \ldots, 4 \\ a^{+j} &\to \phi^{j-4}, && \text{for } j = 5, \ldots, 8 \\ S^a &\to \chi^a_{J=1/2} \end{aligned} \tag{5.6}$$

$AdS_5 \times S^5/\mathbf{Z_k}$ case

Here we have 8 scalars x^I and 8 wordsheet Majorana fermions (θ_1, θ_2) both are massives and can be decomposed under $SO(4)_1 \times SO(4)_2$ as

Fields
$x^I = (\overrightarrow{x}, \overrightarrow{y}) \to (\overrightarrow{x}, z_1, z_2)$
$\theta \equiv \frac{1}{\sqrt{2}}(\theta_1 + i\theta_2) \to \left(\chi^\alpha, \xi^{\dot\alpha}\right)$

(5.7)

Orbifold action	
$g: \overrightarrow{x} \to \overrightarrow{x}, \quad z^m \to \omega z^m$	(5.8)
$g: \chi^\alpha \to \chi^\alpha, \quad \xi^{\dot\alpha} \to \Omega^{\dot\alpha}_{\dot\beta}\xi^{\dot\beta}$	

with α and $\dot\alpha$ are spinor indices of $SO(4)$. We can also combine $\xi^{\dot 1}\ \overline{\xi}^{\dot 2}$ into a Dirac spinor ξ and $\overline{\xi}^{\dot 1}\ \xi^{\dot 2}$ into its conjugate $\overline{\xi}$ and analogously for χ and $\overline{\chi}$.

Now let us identify the bosonic and fermionic operators

Bosonic c.op	*Fermionic c.op*
$\overrightarrow{a}^{+}_{n} \to \overrightarrow{x}$	$\chi^{+}_{n}, \quad \overline{\chi}^{+}_{n}$
$\alpha^{+m}_{n(q)} \to z^m$	$\xi^{+}_{n(q)}$
$\overline{\alpha}^{+m}_{n(-q)} \to \overline{z}^m$	$\overline{\xi}^{+}_{n(-q)}$

(5.9)

Acting the fermionic zero mode oscillators to the light cone vacua and projecting onto $\mathbf{Z_k}$ invariant states, one fills out N=2 gravity and tensor supermultiplets of the plane wave background. The action of the bosonic oscillators on these gives rise to a whole tower of multiplets.

Gauge Theory

Before the orbifold: $(N \times N)$ matrices
Gauge field: A_μ
Complex scalar: $Z = \frac{1}{\sqrt{2}}\left(X^4 + iX^5\right)$
Complex scalar: $\phi^m = \left(\phi^1, \phi^2\right) \equiv \frac{1}{\sqrt{2}}\left(X^6 + iX^7, X^8 - iX^9\right)$
fermion: χ
fermion: ξ

(5.10)

After the orbifolds: $(kN \times kN)$ matrices
$\mathcal{S}\mathcal{A}_\mu\mathcal{S}^{-\infty} = \mathcal{A}_\mu$
$\mathcal{S}\mathcal{Z}\mathcal{S}^{-\infty} = \mathcal{Z}$
$\mathcal{S}\oplus^{\Updownarrow}\mathcal{S}^{-\infty} = \omega\oplus^{\Updownarrow}$
$\mathcal{S}\chi\mathcal{S}^{-\infty} = \chi$
$\mathcal{S}\div\mathcal{S}^{-\infty} = \omega\div$

(5.11)

with $S = \text{diag}\left(1, \omega^{-1}, \omega^{-2}, \ldots, \omega^{-k+1}\right)$, each block being proportional to the $N \times N$ unit matrix.

The operator $\leftrightarrow$ state correspondence is given by

- $\Delta - J = 0$

$$\frac{1}{\sqrt{kJ}N^{J/2}} Tr[S^q \mathcal{Z}^{\mathcal{J}}] \leftrightarrow |\prime, \surd^{+}\rangle_{\text{II}}, \qquad (5.12)$$

$$(q = 0, 1, \ldots, k-1) \qquad (5.13)$$

- $\Delta - J = 1$

$$Tr[S^q \mathcal{Z}^{\mathcal{J}}\mathcal{D}_\mu \mathcal{Z}] \leftrightarrow a^{+\mu}_0 |0, p_+\rangle_q \qquad (5.14)$$

$$Tr[S^q \mathcal{Z}^{\mathcal{J}}\chi_{\mathcal{J}=\infty/\in}] \leftrightarrow \chi^{+}_0 |0, p_+\rangle_q \qquad (5.15)$$

$$Tr[S^q \mathcal{Z}^{\mathcal{J}}\overline{\chi}_{\mathcal{J}=\infty/\in}] \leftrightarrow \overline{\chi}^{+}_0 |0, p_+\rangle_q \qquad (5.16)$$

- $\Delta - J = 2$

Operators corresponding gto higher string states on the pp wave orbifold arise as follows, with a position dependent phase factor in the trace,

$$\sum_{l=1}^{J} Tr[S^q \mathcal{Z}^{\ddagger}\mathcal{D}_\mu \mathcal{Z}\mathcal{Z}^{\mathcal{J}-\ddagger}\mathcal{D}_\nu \mathcal{Z}]]^{\frac{\in\pi\rangle\ddagger}{\mathcal{J}}\backslash} \qquad (5.17)$$

$$\leftrightarrow a^{+\mu}_n a^{+\nu}_{-n} |0, p_+\rangle_q \qquad (5.18)$$

$$\sum_{l=1}^{J} Tr[S^q \mathcal{Z}^{\ddagger}\oplus^{\nabla}\mathcal{Z}^{\mathcal{J}-\ddagger}\overline{\oplus}^{f}]]^{\frac{\in\pi\rangle\ddagger}{\mathcal{J}}\backslash(\text{II})} \qquad (5.19)$$

$$\leftrightarrow \alpha^{r+}_{n(q)}\alpha^{-s+}_{-n(q)} |0, p_+\rangle_q \qquad (5.20)$$

with the rule:

$\mathcal{D}_\mu \mathcal{Z}, \quad \chi_{\mathcal{J}=\infty/\in}, \quad \overline{\chi}_{\mathcal{J}=\infty/\in}$	$\to e^{\frac{2\pi i l}{J}n}$
$\oplus^{\Updownarrow}, \quad \div_{\mathcal{J}=\infty/\in}$	$\to e^{\frac{2\pi i l}{J}n(q)}$
$\overline{\oplus}^{m}, \quad \overline{\div}_{J=1/2}$	$\to e^{\frac{2\pi i l}{J}n(-q)}$

(5.21)

VI. CONCLUSION

In these notes we have considered a Penrose limit of type IIB string theory on $AdS_5 \times S^5$ and $AdS_5 \times S^5/\mathbf{Z_k}$ whose dual field theory is $\mathcal{N} = \triangle$ and $\mathcal{N} = \in$ SYM respectively. This corresponds to looking at operators in the dual field theory with large $J \sim \sqrt{N}$ and finite $\Delta - J$.

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