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UV/IR Mixing, Noncommutative instabilites and Closed Strings

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Abstract

The leading UV/IR mixing effects on noncommutative field theories modify the dispersion relations and, depending on their sign, may cause the appearance of unstable modes. For noncommutative gauge theories on D-branes, this phenomenon is able to capture important information about the closed string spectrum of the parent string theory. We analyse noncommutative D-branes on nonsupersymmetric orbifolds and twisted circle backgrounds. We find that the sign of the leading UV/IR mixing effects is governed by the mass gap between the lowest modes in the NSNS and RR closed string towers. For noncommutative D3-branes at orbifolds we obtain an stronger result: a one to one correspondence between noncommutative instabilities and closed string tachyons.

I. INTRODUCTION

Several arguments suggest the relevance of noncommutative geometry for the description of space-time at short distances. It is thus of importance to study the implications of a noncommutative generalisation of space-time on the dynamics of field theories. In order to explore this issue, most works have considered the simple deformation of $\mathbf{R^n}$

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}\,, \tag{1.1}$$

where $\theta^{\mu\nu}$ is an antisymmetric matrix with constant entries. The reason to focus in (1.1) is that explicit calculations can be done for field theories living on such spaces. The algebra (1.1) implies uncertainty relations in space-time and therefore the non decoupling of ultraviolet and infrared degrees of freedom. This mixing between UV and IR has drastic consequences in the nonplanar sector of field theories: It was shown in[1] that it leads to the appearance of new infrared divergences.

The relations (1.1) are naturally realized on the world-volume of D-branes in a constant B-field background, where $\theta^{\mu\nu} \sim 1/B_{\mu\nu}$. Nonplanar field theory diagrams can then be related to nonplanar string diagrams. This suggests an important role of closed strings in the understanding of UV/IR mixing[1,2]. At a more basic level, we can expect that if (1.1) is able to capture relevant aspects of quantum gravity its effects should know about the closed string sector (although the decoupling of closed strings *does not* fail in the noncommutative field theory limit[3]).

The leading IR divergences induced by UV/IR mixing, modify the dispersion relations of noncommutative field theories as follows

$$E^2 = \vec{p}^2 - c\,\frac{g^2}{\tilde{p}^2}\,, \tag{1.2}$$

where g is the coupling constant, $\tilde{p}^\mu = \theta^{\mu\nu}p_\nu$ and c is a model dependent constant. For noncommutative gauge theories $c \sim N_b - N_f$, with N_b and N_f the number of bosonic and fermionic degrees of freedom in the adjoint representation. This effect is absent in supersymmetric theories since then $c = 0$[4]. When $N_b > N_f$, (1.2) turns the low momentum modes unstable[5,6].

The appearance of instabilities linked to the absence of supersymmetry is reminiscent of a sim-

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ilar phenomenon in string theory. Indeed, modular invariance relates the UV and IR contributions to the torus partition function. As a consequence the absence of supersymmetry generically implies the presence of tachyons in the closed string spectrum[7]. It is then natural to wonder if there can be a relation between noncommutative instabilities and closed string tachyons for those noncommutative theories that can be embedded in string theory[8]. This question was answered affirmatively in[9] for gauge theories associated to noncommutative D-branes in nonsupersymmetric orbifolds, and in[10] for D-branes in twisted circle backgrounds[11]. We will present here a resumé of those results.

II. OPEN WILSON LINES VERSUS CLOSED STRINGS

In this section we will consider noncommutative D3-branes at $\mathbf{C^3/Z_N}$ orbifolds. A $\mathbf{C^3/Z_N}$ orbifold acts with twist $(a_1, a_2, a_3, a_4)/N$ and $(b_1, b_2, b_3)/N$ on $SO(6)$ spinors and vectors respectively. The integers a_α are subject to $\sum a_\alpha = 0(\mathrm{mod}N)$ and are related to b_l by $b_1 = a_2+a_3$, $b_2 = a_1+a_3$, $b_3 = a_1 + a_2$. The gauge theory on n D3-branes placed at the fixed point of the orbifold has gauge group $G = \otimes_{i=1}^N U(n_i)$, where $\sum n_i = n$. The coupling constants of all gauge group factors coincide. The matter content is given by $(fund_i, antifund_{i+a_\alpha})$ Weyl fermions and $(fund_i, antifund_{i+b_l})$ complex scalars. Both the gauge theory on the D3-branes and the closed string spectrum are supersymmetric if at least one $a_\alpha = 0(\mathrm{mod}N)$.

Turning on a B-field background on two of the spatial directions of the D3-branes will render the world-volume noncommutative, *i.e.* $[x^1, x^2] = i\theta$. In the generic nonsupersymmetric case the noncommutative gauge theory presents a complicated pattern of UV/IR mixing effects. The leading infrared contribution to the nonplanar polarization tensor affects only $U(1)_i \in U(n_i)$ degrees of freedom* and is non-diagonal in group labels indices. However the linear combinations $B_\mu^{(k)} = \frac{1}{\sqrt{N}} \sum e^{2\pi i \frac{jk}{N}} \mathrm{Tr} A_\mu^{(j)}$ diagonalize it, with the result[9]

$$\Pi_k^{\mu\nu} = \epsilon_k \, \frac{g^2}{\pi^2} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \,. \tag{2.1}$$

*The theories we are considering have in general mixed anomalies. However when noncommutativity is switched on the anomaly only affects $U(1)$ modes with $\tilde{p} = 0$[12], while for $\tilde{p} \neq 0$ the anomaly vanishes[13]. We will always consider the latter case.

The quantities ϵ_k, which play an analogous role to c in (1.2), have a simple expression in terms of the orbifold twist parameters

$$\epsilon_k = 2 \left(1 - \sum_{\alpha=1}^{4} \cos \frac{2\pi a_\alpha k}{N} + \sum_{l=1}^{3} \cos \frac{2\pi b_l k}{N} \right) . \tag{2.2}$$

Remarkably, these quantities can be rewritten in terms of the masses of four low lying closed string modes in the NSNS k^{th} twisted sector of the orbifold background

$$\epsilon_k = -16 \prod_{\alpha=1}^{4} \sin \frac{\pi \alpha' m_\alpha^2}{2} \,. \tag{2.3}$$

These masses satisfy the following properties: *i)* among them is the lowest mode in the NSNS k^{th} twisted sector; *ii)* $-1 \leq \alpha' m_\alpha^2 < 2$; *iii)* only one of the m_α^2 can be negative (see[9] for details). This implies a direct relation between the sign of ϵ_k and the presence or not of tachyons in the k^{th} twisted sector of the parent string theory: $\epsilon_k > 0$ if the k^{th} twisted sector contains tachyons; $\epsilon_k = 0$ only if the k^{th} twist is supersymmetry-preserving; $\epsilon_k < 0$ when the twisted sector is nonsupersymmetric but does not include tachyons.

This remarkable relation between noncommutative instabilities and closed string tachyons calls for a derivation purely in terms of closed strings of the leading UV/IR mixing effects. As a first step, let us come back to the origin of UV/IR mixing, which is in the uncertainty relations derived from (1.1): $\Delta x^1 \Delta x^2 \geq \theta$ in our case. It would seem that the natural degrees of freedom of noncommutative theories must satisfy the previous uncertainty relations. Contrary to this expectation, noncommutative field theories are formulated using local fields. This puzzle is solved by studying the effective action of the theory. It has been shown both for scalar[14] and gauge theories[15,8,16] that the 1-loop nonplanar effective action, including contributions from all the N-point functions, can be rewritten in terms of straight open Wilson line operators[17]. Straight open Wilson line operators exhibit the desired behaviour since their momentum, p, is correlated with their transversal extent $\tilde{p}$,

$$\widetilde{W}(p) \;=\; \mathrm{Tr} \int d^4x \; P_* \left(e^{i\,g \int_0^1 d\sigma \, \tilde{p}^\mu A_\mu (x+\tilde{p}\,\sigma)} \right) * e^{ipx} \,. \tag{2.4}$$

For the orbifold gauge theories above, the gauge invariant piece of the 1-loop effective action containing (2.1) has the simple expression[9]

$$\Delta S = \frac{1}{2\pi^2}\sum_{k=0}^{N-1}\epsilon_k \int \frac{d^4p}{(2\pi)^4}\,\frac{1}{\tilde{p}^4}\,W^{(N-k)}(p)\,W^{(k)}(-p)\,, \tag{2.5}$$

where $W^{(k)} = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{2\pi i \frac{jk}{N}}\widetilde{W}^{(j)}$, and $\widetilde{W}^{(j)}$ denotes (2.4) with the vector field belonging to the j^{th} gauge group factor.

Several facts suggest the interpretation of (2.5) in terms of a closed string exchange between D-branes. This action seems to know about the different closed string twisted sectors since the quantities ϵ_k measure an independent property of each sector. Closed string modes in the k^{th} twisted sector couple naturally to linear combinations of field theory operators such as those that define B^k_μ and W^k[18]. Moreover, it has been shown in[19] that closed strings couple to straight Wilson line operators on noncommutative D-branes. In order to complete the argument, we only need to show that the term $1/\tilde{p}^4$ in (2.5) can be related to a closed string propagator.

In the absence of B-field, scalar closed string modes couple to the brane tension $T_{D3} \sim \mathrm{Tr}\,\mathbf{1}/\alpha'^2$ at leading order in α'. When $B \neq 0$ the trivial field theory operator $\mathrm{Tr}\,\mathbf{1}$ gets promoted to the open Wilson line operator (2.4). Hence, at leading order in α', the contribution to the D3-brane effective action from the emission, propagation and posterior absorption of a scalar closed string mode φ in the k^{th} twisted sector is

$$\Delta S \sim \int \frac{d^4p}{(2\pi)^4}\,W^{(N-k)}(p)W^{(k)}(-p)\,f(\tilde{p},u)\,, \tag{2.6}$$

where the function $f(\tilde{p},u)$ denotes the closed string propagator

$$f(\tilde{p},u) = \alpha'^{-\frac{d+2}{2}}\int \frac{d^dv}{(2\pi)^d}\frac{e^{ivu}}{v^2+\tilde{p}^2+(2\pi\alpha' m_\varphi)^2}\,. \tag{2.7}$$

d is the number of dimensions transverse to the D-brane where the twisted field φ can propagate: $d = 0,2,4,6$ depending on the particular $\mathbf{C^3/Z_N}$ orbifold. We have defined $v = 2\pi\alpha' p_\perp$, with $p_\perp$ the transversal momentum to the D-brane. u has been introduced in order to have a well defined closed string propagator; it has the interpretation of an infrared regulator from the point of view of the field theory. In the denominator we have used the relation between open (η) and closed (g) string metrics $g^{-1} = \eta^{-1} - \theta\eta\theta/(2\pi\alpha')^2$[20], and discarded terms suppressed by two α' powers. m_φ is the mass of φ. The factor of α' in front of the integral can be obtained just by dimensional analysis.

We want to extract from (2.6) a contribution to the noncommutative field theory effective action. We need to take the limit $\alpha' \to 0$. In this limit f diverges due to the negative α' power in front of the integral. However we should notice the following. If we had done the $\alpha' \to 0$ limit of the standard annulus diagram associated to each non-planar N-point function, we would had obtained a result $\mathcal{O}(\alpha'^0)$ whose leading IR contribution should reproduce the corresponding term in (2.5). The question is then whether we can directly define an $\mathcal{O}(\alpha'^0)$ contribution from (2.6), regarding as artifacts other α' powers. We observe that $\tilde{p}$ acts as an infrared regulator for the integral in (2.7). Therefore, for $\tilde{p} \neq 0$, we can expand the integral in powers of $(2\pi\alpha' m_\varphi)^2 \sim \alpha'$ to the desired order. At $\mathcal{O}(\alpha'^0)$ we obtain

$$f(\tilde{p},u)|_{\mathcal{O}(\alpha'^0)} \sim \int \frac{d^dv}{(2\pi)^d}\frac{e^{ivu}}{(v^2+\tilde{p}^2)^{\frac{d}{2}+2}} \sim_{u\to 0} \frac{1}{\tilde{p}^4}\,. \tag{2.8}$$

After removing the field theory infrared regulator u we recover $1/\tilde{p}^4$, independently of d. The possibility to relate $1/\tilde{p}^4$ to a closed string propagator does not mean that the decoupling of closed strings fails in the noncommutative field theory limit, since the IR singularities do not have kinetic part. Hence they do not force the introduction of additional degrees of freedom.

In (2.6) we have considered the exchange of a single closed string mode. Any closed string mode able to couple to W^k will contribute the same $1/\tilde{p}^4$ up to a numerical factor depending on m_φ and the disk amplitude of φ with boundary conditions on the brane. The coefficients ϵ_k that appear in (2.5) will be thus a collective effect of the closed string towers. However, we know from (2.3) that ϵ_k is determined by a finite set of low lying string modes and, moreover, its sign just depends on the presence of tachyons. To reconcile these two facts, notice that modes in the NSNS and RR sectors contribute with opposite sign to the exchange between D-branes. In addition, when the theory is supersymmetric we have $\epsilon_k = 0$, implying that the contribution from both towers must cancel. This suggests to consider ϵ_k as a measurement of the misalignment between the NSNS and RR towers. Tachyons can only belong to twisted NSNS sectors, while the lightest RR twisted modes are always massless in orbifold backgrounds. It seems then consistent that the presence of tachyons is linked to the sign of the mentioned misalignment inside each twisted sector.

III. GENERALISATION: D-BRANES ON TWISTED CIRCLES

We would like to know whether the relation we have obtained between UV/IR mixing effects and properties of the closed string spectrum can be applied to more general situations. An interesting example to analyse is that of D-branes on twisted circle backgrounds. These are $\mathbf{R^9} \times \mathbf{S^1}$ space-times where shifts along the circle are combined with rotations on several 2-planes[11]. We will only consider rotations which coincide with those defining a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold. When the radius of the circle, R, is send to infinity we recover Type II string theory on $\mathbf{R^{10}}$. For $R = 0$ the Type IIA(B) twisted circle backgrounds reduce, after T-duality, to the associated $\mathbf{C^3}/\mathbf{Z_N} \otimes \mathbf{R^4}$ Type IIB(A) orbifold model[21].

The spectrum of closed strings on twisted circles was derived in[11]. For the cases we are interested in, the modes with winding number w along the $\mathbf{S^1}$ are closely related to the spectrum of the associated orbifold model in the twisted sector $k = w \text{ mod} N$. In particular, the spectrum of modes with zero momentum along $\mathbf{S^1}$ precisely coincides with that of the orbifold model, except for a positive mass shift due to the winding energy. It is this shift what stabilises otherwise tachyonic modes for sufficiently large radius.

Let us analyse what this implies for the UV/IR mixing effects on noncommutative D-branes in twisted circle backgrounds. The closed string derivation implies that the signs of the leading UV/IR mixing effects are linked to the misalignment between the NSNS and RR towers of the parent string theory. We will focus in situations where only closed string modes with zero momentum on $\mathbf{S^1}$ can contribute (see[10] for details). Since the winding energy affects equally the NSNS and RR towers, we should expect that the signs of the leading UV/IR mixing effects are independent of R. Moreover, they should coincide with those of the associated orbifold background.

In order to check these predictions we consider Type IIB D3-branes wrapped on the twisted circle and Type IIA D2-branes transversal to it. The appropriate field theory limits for these branes differ. The intrinsic scale of the D3-brane theory is set by the KK masses $m \sim 1/R$. Hence we should send $\alpha' \to 0$ keeping R fixed. In this regime the winding energy dominates the closed string spectrum and there are no tachyonic modes. Contrary, the natural scale of the D2-brane theory is R/α', governing the mass of strings ending on the brane and with non-zero winding along the circle. The interesting field theory limit in this case is $\alpha' \to 0$ with R/α' fixed. This corresponds to the limit of negligible winding energy, where the closed string spectrum contains tachyons. Therefore these two examples allow us to explore opposite regimes of the closed string background.

An analysis of the gauge theory on noncommutative D2- and D3-branes shows[10] that the signs of the UV/IR mixing effects for both cases are governed by the same quantities ϵ_k which, as we expected, coincide with those of the orbifold background (2.2). The functional dependence of the leading UV/IR mixing terms on $\tilde{p}$ is however different for D2- and D3-branes, and in both cases can be related to a closed string propagator. We found IR finite corrections for D3-branes and IR divergent ones for D2-branes. For D2-branes there is a one to one correspondence between noncommutative instabilities and closed string tachyons, as it was the case for D3-branes at orbifolds. It is tempting to speculate that a direct relation between noncommutative instabilities associated to IR divergences and closed string tachyons might hold in general. For D3-branes at twisted circles we have a weaker but still interesting result. UV/IR mixing effects are still governed by the lowest modes in the associated closed string sector. Moreover destabilising UV/IR mixing effects are absent for those sectors related to closed strings which are stable for any radius.

At a more fundamental level, we are seeing that the gauge theory on the world-volume of D-branes contains very specific information about the closed string spectrum. This information is somehow hidden in an ordinary situation, but becomes manifest through UV/IR mixing when noncommutativity is turned on. It would be very interesting to determine how much information about string theory spectrum can be extracted in this way from the low energy theory on D-branes.

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