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Aspects of Inflation in Brane Physics

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Abstract

We study brane effect of the behaviour of universe in inflationary model and analyze the change of slow-roll approximation for an arbitrary inflaton potential. We discuss the modified Friedmann equations and give the analytical expression for the scale factor in the case of chaotic potential which is shown to have an exponential form as expected .

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I. INTRODUCTION

Inflation theory [1] solves most of known problems of standard cosmology and provides a mechanism for the generation of density perturbations which leads to the formation of structures in the universe. In this hot big bang standard model, universe is supposed to undergo a quasi-exponential expansion caused by inflaton vacuum energy.

Recently developments has been achieved in studding inflationary cosmological model in the framework of brane physics. This issue, which is motivated by progress in superstring theory [2], has led to new solutions [3] where matter, described by open string modes, lives on a d-dimensional brane while gravity, associated with closed string modes, propagates in an $d^{th}+1$ extra bulk dimension. Since then, diverse inflationary brane models were considered [4]. In most of these models, the main role is played by a scalar field rolling in an inflation potential. The dynamicss of the scalar field was also shown to give a realistic explanation of a dark energy problem[5] and the observed accelerating universe[6] as well as in describing tachyonic inflation[7].

In this paper, we study aspects of cosmological models using brane scenario and give explicit analysis of the induced changes in the cosmological parameters. Recall that the brane world scenario is a 4D effective gravity in the world volume of a 3-brane with a special bridge to 5D Einstein gravity[8]. In this context and following Randall and R.Sundrum (SR)[9], the fundamental Planck scale M_{4+d} in $4+d$ dimensions can be considerably smaller than the 4D effective Planck scale, $M_4 = 1.2\times 10^{19}$ GeV. This remarkable development in cosmology has been shown to have profound consequences on inflationary models of the very early universe. In the present work, we focuss on a 5D SR model ($d=1$) with chaotic brane inflation potentials[10] and study the modifications of slow-roll approximation in the brane scenario, independently of the dynamicss of the bulk assuming the stability of the brane.

The presentation of this work is as follows: In section 2, we review briefly the basis of standard cosmology. In section 3, we study inflationary paradigm and some related models especially chaotic potentials. In section 4, we consider brane world cosmology with particular focus on the scale factor and comparison with known results. Others cosmological quantities are also computed. Last section is devoted to conclusion and perspectives.

II. STANDARD COSMOLOGY

It is well known that classical models of universe are obtained by resolving Friedman equations deduced from Einstein equations describing gravitational field[11]. In the framework of general relativity, these equations are given by:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} \tag{2.1}$$

In standard cosmology, we assume that universe is similar to a perfect fluid with pressure p and density ρ. In this case, the energy-momentum tensor $T_{\mu\nu}$ is of the form:

$$T_{\mu\nu} = p g_{\mu\nu} + (p+\rho)u_\mu u_\nu \tag{2.2}$$

where $u^\mu = \frac{ds}{d\tau}$ is quadri-vector velocity..

To have dynamicsal solutions, one has to consider the Robertson-Walker-Friedmann (FRW) metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\varphi^2)\right] \tag{2.3}$$

$a(t)$ is the scale factor describing the physical size of the universe. Note in passing that the mathematical form of $a(t)$ allows us to analyze the behavior of the dynamicss of universe over time. The constant k is related to the spatial curvature, with k negative, null or positive corresponding to an open, flat or close universe respectively. By injecting the metric (2.3) in Einstein equations(2.1), one obtains the following Friedmann equations describing the expansion of universe[11]:

$$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\rho - \frac{k}{a^2} \tag{2.4}$$

$$\dot\rho + 3H(\rho+p) = 0 \tag{2.5}$$

here we have introduced the Hubble constant $H = \dot a/a$.

An equivalent form of the above equations containing the second derivative of the scale factor $a(t)$ reads as:

$$\frac{\ddot a}{a} = -\frac{4\pi}{3m_{\rm Pl}^2}(\rho + 3p), \tag{2.6}$$

where we have set $k = 0$ for simplicity. The different phases of the universe are determined by a judicially choice of the state equation between $\rho(t)$ and $p(t)$. We can thus obtain the radiation or the matter dominated era and finally the scalar matter dominated era describing the inflation.

From this equation, we see that the universe undergoes an accelerated expansion (inflation), when $\ddot a \succ 0$ which leads to $\rho \prec -3p$. In section 4, we shall show that we can have a more restrictive relation between ρ and p in the brane world theory[9].

III. INFLATIONARY FORMALISM AND APPLICATIONS

A. Scalar Field dynamics

The standard model based on Friedmman equations agrees with many observed predictions like the Hubble expansion of the universe, the elements abundance, and the microwave background radiation of $3^\circ k$[5], as well as others indirect tests. However this scenario presents some inefficiencies like horizon and flatness problems[1]. To solve these problems, A.Guth[1] has introduced the idea of inflation which suppose that universe during its evolution have passed an exponential expansion phase. At present, these exists several inflationary models depending on the choice of the mathematical form of the potential[12]. One of the aims of this article is to show how inflationary dynamicss of universe is modified in brane models. We discuss particularly some simplest models of brane world potential[13].

To start, consider a Lagrangian density of a scalar field describing inflation dynamics:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi). \tag{3.1}$$

where $\phi(x,t)$ is a scalar field and $V(\phi)$ is the potential. The corresponding energy-momentum tensor take the following form for a *perfect fluid*,

$$T^{\mu\nu} = diag(\rho, p, p, p). \tag{3.2}$$

Thus, one can easily derive, for a diagonal metric, the following expression for p and ρ:

$$p = \frac{1}{2}\dot\phi^2 - V(\phi), \qquad \rho = \frac{1}{2}\dot\phi^2 + V(\phi). \tag{3.3}$$

In this expression ϕ is the inflaton that is the scalar field whose the potential $V(\phi)$ is responsible of exponential expansion of universe.

The standard model of cosmology is based on Einstein theory of gravitation. In the inflationary scenario, the Friedmann equations take the form[1]:

$$H^2 = \frac{8\pi}{3m_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi),\right] \tag{3.4}$$

where H is the Hubble constant and the dot is the derivative with respect to time t. The dynamics of the universe is determined once the equation of state between the energy density $\rho(t)$ and the pressure $p(t)$ is defined. In the framework of inflationary theory, the density ρ and the pressure p are given by equations (3.3). The substitution of eqs(3.3) in (2.4)and (2.5) leads to the inflationary dynamics equations where we neglect the curvature term K/a^2:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \tag{3.5}$$

To calculate physical quantities like scale factor or perturbation spectrum, one has to solve the above equations(3.4) and (3.5) for some specific potential. To do so, some approximation must be introduced.

B. Slow-roll approximation

The inflationary dynamics requires that the inflaton field deriving inflation moves away from the false vacuum and slowly rolls down to the minimum of its effective potential[1]. In this scenario, the scalar field and the Hubble parameter H was initially very large. Thus, the scale factor of the universe growth rapidly,

The inflation condition $\ddot{a} \rangle 0$ leads to $\rho + 3p \langle 0$ ($\dot{\phi}^2 \langle V(\phi)$), thus the "**Slow-Roll**" approximation reads

$$\frac{\dot{\phi}^2}{2} \langle\langle V(\phi), \qquad \ddot{\phi} \langle\langle 3H\dot{\phi} \tag{3.6}$$

Consequently, in the framework of slow roll approximation the eqs(3.4) and (3.5) become:

$$H^2 \simeq \frac{8\pi}{3m_{pl}^2}V(\phi) \tag{3.7}$$

$$3H\dot{\phi} \simeq -V'(\phi) \tag{3.8}$$

To discuss the validity of the slow roll approximation(SRA), one has to define the two new parameters ϵ and η,

$$\epsilon(\phi) = \frac{M_p^2}{16\pi}(\frac{V'}{V})^2, \tag{3.9}$$

$$\eta(\phi) = \frac{M_p^2}{8\pi}(\frac{V''}{V}). \tag{3.10}$$

The "SRA" is then valid if $\epsilon \langle\langle 1$ and $|\eta| \langle\langle 1$. We note, in passing that the inflationary phase ends when ϵ and $|\eta|$ are approximately of order unity. Another very important quantity indicating the physical behaviour of inflation is the number of e-folding indicating the growing of the size of universe defined as:

$$N = \ln\frac{R_f}{R_i} = \int_{t_{i=}}^{t_f} H(t)dt \tag{3.11}$$

where the subscripts i and f denote the quantities at the beginning and at the end of inflation respectively.

We can also express the e-folding number of this parameter in term of Hubble constant

$$a(t) = a_0 \exp(Ht) \Leftrightarrow \ln(\frac{a(t)}{a_0}) = Ht,$$

$$\mathcal{N} \equiv \ln(\frac{a(t_{final})}{a(t_{initial})}) = \int_{t_i}^{t_f} Hdt \tag{3.12}$$

or in term of the potential

$$\mathcal{N} = \int_{t_i}^{t_f} Hdt = \int_{t_i}^{t_f} H\frac{dt}{d\phi}.d\phi$$
$$= \int_{t_i}^{t_f}\frac{H}{\dot{\phi}}.d\phi = -\frac{8\pi}{M_p^2}\int_{\phi_i}^{\phi_f}\frac{V}{V'}.d\phi.$$

from the equations (3.7-8), one can obtain

$$\frac{3\dot{\phi}}{H} = \frac{3H\dot{\phi}}{H^2} = -\frac{V'}{8\pi V}(3M_p^2),$$
$$\Rightarrow \frac{\dot{\phi}}{H} = -\frac{M_p^2}{8\pi}(\frac{V'}{V}) \tag{3.13}$$

thus,

$$\mathcal{N} \simeq \frac{8\pi}{M_p^2}\int_{\phi_f}^{\phi_i}\frac{V}{V'}.d\phi. \tag{3.14}$$

As an application of the precedent formalism, we shall consider in the following the chaotic inflation witch have the polynomial potential:

$$V(\phi) = \frac{1}{2}m^2\phi^2 \tag{3.15}$$

where m is scalar field mass (inflaton).

Thus, the equations (3.7) and (3.8) are transformed as:

$$H^2 \simeq \frac{4\pi m^2\phi^2}{3m_{pl}^2}, \tag{3.16}$$

$$3H\dot{\phi} + m^2\phi \simeq 0 \tag{3.17}$$

which can be easily solved as

$$H = \sqrt{\frac{4\pi m^2\phi^2}{3m_{pl}^2}} \simeq \frac{2m\phi}{m_{pl}}\sqrt{\frac{\pi}{3}} \tag{3.18}$$

and

$$\phi(t) \simeq \phi_i - \frac{mm_{pl}}{2\sqrt{3\pi}}t \tag{3.19}$$

$\phi_i = \phi(t=0)$.

To calculate the scale factor $a(t)$, one has to integrate equation(3.18), and obtain the following form:

$$a(t) \simeq a_i \exp 2\sqrt{\frac{\pi}{3}}\frac{m}{m_{pl}}\left[\phi_i t - \frac{mm_{pl}}{4\sqrt{3\pi}}t^2\right] \tag{3.20}$$

$a_i = a(t=0)$.

The form of the scale factor $a(t)$ shows an exponential form, which is conform with the inflationary hypothesis. We can also obtain the "SR" parameters as:

$$\epsilon = \eta = \frac{m_{pl}^2}{4\pi\phi^2} \tag{3.21}$$

As signaled before, the inflation ends when $\epsilon = 1$, which implies that: $|\phi| = \frac{m_{pl}}{\sqrt{4\pi}}$. At this moment the scale factor presents an inflection point allowing us to calculate t_f; the time at which the inflation ends:

$$\ddot{a}=0 \Rightarrow t = t_f = \frac{2\sqrt{3\pi}}{mm_{pl}}\left[\phi_i - \frac{m_{pl}}{\sqrt{4\pi}}\right] \tag{3.22}$$

After the inflation period, the universe undergoes an oscillating phase around $|\phi|$, in which the elementary particles will be created.

Another important parameter of inflation is the e-folding number. In chaotic inflation it takes the form:

$$N = \ln\frac{a(t)}{a_i} = 2\sqrt{\frac{\pi}{3}}\frac{m}{m_{pl}}(\phi_i t - \frac{mm_{pl}}{4\sqrt{3\pi}}t^2) \tag{3.23}$$

at the end of inflation it becomes:

$$N = 2\pi\left(\frac{\phi_i}{m_{pl}}\right)^2 - \frac{1}{2} \tag{3.24}$$

if $N \geq 70$ one must have $\phi_i \geq 3m_{pl}$.

IV. BRANE WORLD INFLATION

A. Modified Friedmann equations

By considering Einstein's equations in 5d bulk, with a cosmological constant as source, and the matter fields are confined to the 3-brane, Shiromizu et al.[14] have shown that the 4d Einstein equations induced on the brane are written as

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left(\frac{8\pi}{M_4^2}\right)T_{\mu\nu} + \left(\frac{8\pi}{M_5^3}\right)^2 \pi_{\mu\nu} - E_{\mu\nu}, \tag{4.1}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter on the brane and $\pi_{\mu\nu}$ an tensor quadratic in $T_{\mu\nu}$. The term $E_{\mu\nu}$ describes the effect of bulk graviton degrees of freedom on brane dynamics.

The effective cosmological constant on the brane Λ_4 is given in term of the 5d bulk one Λ and the 3-brane tension λ as

$$\Lambda_4 = \frac{4\pi}{M_5^3}\left(\Lambda + \frac{4\pi}{3M_5^3}\lambda^2\right), \tag{4.2}$$

where the 4d and 5d Planck scales are related as

$$M_4 = \sqrt{\frac{3}{4\pi}}\left(\frac{M_5^2}{\sqrt{\lambda}}\right)M_5. \tag{4.3}$$

On the other hand, Binetruy et al.[8] have given a generalization of Friedmann equation on the brane, and show that

$$H^2 = \frac{\Lambda_4}{3} + \left(\frac{8\pi}{3M_4^2}\right)\rho + \left(\frac{4\pi}{3M_5^3}\right)^2\rho^2 + \frac{\mathcal{E}}{a^4}, \tag{4.4}$$

where $\mathcal{E}$ is an integration constant related to the tensor $E_{\mu\nu}$, indicating a bulk graviton influence on the brane. This term was also shown to be attached to a sort of dark radiation or matter[14] and can be observed in microwave background radiation. This allow us to have a lower limit on $|\mathcal{E}|$ and we shall neglect it in the following. To have a simplified brane equations we assume also that $\Lambda \approx -4\pi\lambda^2/3M_5^3$ so that Λ_4 is negligible.

We observe that the crucial correction is due to a quadratic term in the density, This will modify the expansion dynamics of the universe for densities $\rho \gtrsim \lambda$.

Consequently, generalized Friedmann equation on the brane ($\Lambda_4 = 0$ and $\mathcal{E} = \prime$) can now be written as:

$$H^2 = \frac{8\pi}{3M_4^2}\rho\left[1 + \frac{\rho}{2\lambda}\right]. \tag{4.5}$$

We note that in the limit $\lambda \to \infty$, one can discover the standard four-dimensional Friedmann equation (2.4).

B. Slow-roll inflation on the brane

In this paragraph, we shall see how the "SRA" is modified in the brane world, in order to solve the brane dynamics for some potential. We consider a model with an energy-momentum tensor $T_{\mu\nu}$ on the brane dominated by a scalar field ϕ,with a self-interaction potential $V(\phi)$. The scalar field satisfies the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,, \tag{4.6}$$

If we apply now the inflation condition $\ddot{a} > 0$ to the modified Friedmann equations, we can derive a stronger condition for inflation:

$$\ddot{a} > 0 \quad \Rightarrow \quad p < -\left[\frac{\lambda + 2\rho}{\lambda + \rho}\right]\frac{\rho}{3}\,. \tag{4.7}$$

which becomes in the limit $\rho/\lambda \to \infty$: $p < -\frac{2}{3}\rho$. We see that this later is more restrictive than the standard one(p$\prec -\frac{\rho}{3}$).

Thus in the slow-roll approximation, the energy density is dominated by the self-interaction energy of the scalar field so that:

$$H^2 \simeq \left(\frac{8\pi}{3M_4^2}\right) V \left[1 + \frac{V}{2\lambda}\right]\,, \tag{4.8}$$

$$\dot{\phi} \simeq -\frac{V'}{3H}\,, \tag{4.9}$$

The symbol '$\simeq$' indicates that we have taking into account the slow-roll approximation. The presence of the term $\left[1 + \frac{V}{2\lambda}\right]$ represents a brane-modification to the standard slow-roll expression. For $V \gg \lambda$, the brane effect affects universe behaviour.

Define now the two slow-roll parameters [14], as:

$$\epsilon \equiv \frac{M_4^2}{16\pi}\left(\frac{V'}{V}\right)^2 \left[\frac{2\lambda(2\lambda + 2V)}{(2\lambda + V)^2}\right]\,, \tag{4.10}$$

$$\eta \equiv \frac{M_4^2}{8\pi}\left(\frac{V''}{V}\right)\left[\frac{2\lambda}{2\lambda + V}\right]\,. \tag{4.11}$$

to be conform with the slow-roll approximation, one must have $\max\{\epsilon, |\eta|\} \ll 1$.

The number of e-folds:

$$N = \int_{t_i}^{t_f} H dt \tag{4.12}$$

in the slow-roll approximation

$$N \simeq -\frac{8\pi}{M_4^2}\int_{\phi_i}^{\phi_f} \frac{V}{V'}\left[1 + \frac{V}{2\lambda}\right] d\phi\,. \tag{4.13}$$

We have thus two physical limit to be considered. At low energies, $(V \ll \lambda)$, the slow-roll parameters take the standard form, whereas at high energies, $V \gg \lambda$, the extra contribution to the Hubble expansion dominates and the new factors in square brackets become of order λ/V. In this case the brane effects favorites more the slow-roll condition for inflation for a given potential.

Finally, we note that for $V \gg \lambda$, Equation (4.13) can be rewritten as

$$N \simeq -(128\pi^3/3M_5^6)\int_{\mathrm{i}}^{\mathrm{f}} (V^2/V')d\phi. \tag{4.14}$$

C. Chaotic inflation on the brane

As an application of the above formalism, we shall now apply it for a simplest model especially chaotic inflation. In the brane world approximation $(V \gg \lambda)$, the equations (4.8) become:

$$H^2 \simeq \left(\frac{8\pi}{3M_4^2}\right)\frac{V^2}{2\lambda} = \left(\frac{4\pi}{3M_5^3}\right)V^2, \tag{4.15}$$

$$\Rightarrow \dot{\phi} \simeq -\frac{V'}{3\left(\frac{4\pi}{3M_5^3}\right)V} \cong -\frac{V'}{V}\left(\frac{M_5^3}{4\pi}\right) \tag{4.16}$$

which lead, for the potential $V(\phi) = \frac{1}{2}m^2\phi^2$, to:

$$\phi = (\phi_i^2 - \frac{M_5^3}{\pi}t)^{\frac{1}{2}} \tag{4.17}$$

and

$$a(t) = a_i \exp[\left(\frac{2\pi m^2}{3M_5^3}\right)(\phi_i^2 t - \frac{M_5^3}{2\pi}t^{\,2})] \tag{4.18}$$

The Slow Roll parameters are:

$$\epsilon = \eta = \frac{3M_5^6}{8\pi^2 m^2}\frac{1}{\phi^4} \tag{4.19}$$

We remark that in the limit $\phi \to \infty$, $\epsilon = \eta$ decrease rapidly to 0. and the slow roll approximation is more valid in brane world scenario.

V. CONCLUSION AND PERSPECTIVES

In this paper, we have studied inflationary theory using brane world cosmology. We have reconsidered the brane effect modifications brought to Friedmann equations governing the dynamics of scalar field. Extra term depends on the ratio of the density of matter ρ and the brane tension λ. Slow roll approximation is shown to simply enormously Friedman equation which depends only on the ratio of the potential V and the tension λ.

At low energies ($V \ll \lambda$), the slow-roll parameters take the standard form, indicating that the effect of brane is neglected, whereas at high energies, $V \gg \lambda$, the extra contribution to the Hubble expansion dominates and a new factor in square brackets becomes of order λ/V. In this case, the condition for slow-roll inflation becomes more favorable by brane effects for a given potential. We have also calculated the scale factor for a chaotic potential in brane world formalism, and show that it has an exponential form as it should be for an inflationary model. This work may be extended to more general potentials such as inverse power law $\sim \Phi^{-\alpha}$. The perturbation spectrum of this potential still need to be completed as well as comparison with observation. An other issue to be addressed in [15] concerns other type of scalar matter such that the tachyonic field[16] and dark matter[17].

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