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Dissociation of a boosted quarkonium in quark-gluon plasma

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Abstract

I consider the dissociation of a boosted quarkonium in a quark-gluon plasma. This dissociation is due to absorption of a thermal gluon. I discuss the dissociation in terms of the velocity of the quarkonium and the temperature of the quark-gluon plasma. I compare this dissociation rate to the one calculated without including the velocity of the quarkonium.

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In a quark-gluon plasma, the quark or the anti-quark of the quarkonium can be struck by a high energy gluon and the quarkonium can dissociate into other elements [1]. The medium, the quark-gluon plasma, is full of gluons that can cause this dissociation, and this can happen by exciting a color-singlet $\mid q\bar{q}\rangle^{(1)}$ into a color-octet continuum state$\mid q\bar{q}\rangle^{(8)}$ [2]. The quark absorbs energy from the gluonic field of the quark-gluon plasma, but there is a threshold energy that the gluon has to have in order to dissociate the quarkonium in the quark-gluon plasma. For example, in free space, there is a threshold energy of about 850 MeV [3] for the dissociation of the upsilon meson into two highly-energetic bottom quarks. I choose the upsilon meson, Υ, for simplicity, since it is heavy and its binding energy is large compared to other mesons such as the pion or the J/ψ which dissociate fast and at relatively low temperature [4]. Moreover, in an experiment which produces a very high temperature quark-gluon plasma, the J/ψ will break down at a faster rate compared to the Υ and only a few of the J/ψ's will survive. This makes it hard for the experimentalist to detect the J/ψ in order to confirm that a quark-gluon plasma had materialized. On the other hand, the Υ will survive the high temperature of the quark-gluon plasma and therefore it will be easy to detect. Moreover, the upsilon meson is small in size compared to the J/ψ so it needs a higher density plasma for it to dissociate. In this kind of dissociation, we state that the Υ is no longer bound in the singlet state. In quark-gluon plasma the threshold energy will become less than the one in free space. Only gluons with energy exceeding the threshold energy in quark-gluon plasma can dissociate the quarkonium. So the relevant energy density is not just the average energy density, but the energy of the gluons which have an energy higher than the threshold energy. In deconfined matter, such as quark-gluon plasma, we expect gluons in a medium of 200 MeV temperature to have an average energy of 600 MeV [5], which can dissociate the quarkonium.

The effect of screening can also cause the meson to dissociate. A screen of quarks and gluons builds up between the quark and the anti-quark of the quarkonium and the two quarks cannot feel each other's attraction and the quarkonium dissociates. This kind of screening is called Debye screening [6] and I discussed it in a previous work [7]. In regards to screening, once we take the motion of the quarkonium into consideration, the

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dielectric properties of the medium [8], which is the quark-gluon plasma, gets introduced and the physics becomes somewhat involved.

When the heavy quark and anti-quark of the quarkonium are close to each other, asymptotic freedom comes into play, and the binding energy can be derived the same way we derive it for the hydrogen atom. To an approximation, there is a parallelism between the two physics [2].

The wavelength of the gluon that dissociates this state fits the radius of the singlet state of Υ to a good approximation. The S matrix in this case is:

$$S_{fi} = -i \int_{-\infty}^{\infty} dt \langle \text{octet} \mid grE^a \cos\theta \mid \text{singlet} \rangle \quad (1)$$

where E^a is the color electric field and g is the color charge. It is worked out in [2,7] that the dissociation rate of the upsilon meson becomes:

$$\Gamma_{dis} \approx \frac{2}{3}\pi^3 \alpha_s a^2 n \,, \quad (2)$$

where a is the radius of the singlet state of the quarkonium, and n is the number density of gluons in the quark-gluon plasma. Since we are considering thermalization, this dissociation rate can be calculated in terms of the temperature of the quark-gluon plasma. But I will only include the gluons with energy exceeding the threshold energy of dissociation, ω_{min}. For a medium of gluons

$$n = N/V = \frac{1}{2\pi^2} \int_{\omega_{\text{min}}}^{\infty} d\omega \frac{\omega^2}{\exp(\omega/T) - 1} \,, \quad (3)$$

which gives us $\tau_{dis} \approx \frac{4\alpha_s}{3\pi} \frac{m_Q^2}{A}$,

$$A = \sum_{k=1}^{\infty} \left[\left(\frac{T}{k}\right) \omega_{\text{min}}^2 + 2\left(\frac{T}{k}\right)^2 \omega_{\text{min}} + 2\left(\frac{T}{k}\right)^3 \right] \times e^{-k\omega_{\text{min}}/T} \,, \quad (4)$$

where τ_{dis} is the dissociation time of the quarkonium. The minimum temperature required to achieve deconfinement is generally understood to be about 150 to 200 MeV. RHIC, the Relativistic Heavy Ion Collider, is the first collider designed to specifically create this plasma. It may reach a temperature of 500 MeV. CERN is trying to reach a temperature of 1 GeV by colliding heavy nuclei in the Large Hadron Collider. Inserting a temperature of 500 MeV and a threshold energy ω_{min} of 10 MeV into eq.(4), I get a dissociation time of 23 fm/c, which is somewhat comparable to the lifetime of the quark-gluon plasma–a typical lifetime of a quark-gluon plasma is about 10-20 fm/c [9,10]. If I use the CERN 1 GeV temperature, I get $\tau_{\text{dis}} \approx 3$ fm/c. We should remember that I used the threshold energy of 10 MeV as the binding energy of the upsilon quarkonium. This is due to screening, and to the fact that the medium is hot.

In the case of a moving quarkonium, we have to consider the velocity when calculating for the dissociation time. And since the quarkonium is moving in the frame of the gluons, the gluons will be relatively moving in the frame of the quarkonium. So in this case, the momentum, ω, of the gluon will change into $\omega' = \gamma\omega(1 - v\cos\theta)$, where θ is the angle between the velocity vector of the meson and the momentum vector of the gluon, and v is the relative velocity of the quarkonium in the quark-gluon plasma. The new number density of gluons in quark-gluon plasma, in this case, is:

$$n' = \frac{1}{2\pi^2} \int_{\omega_{min}}^{\infty} d\omega \int_{1}^{-1} d(\cos\theta) \frac{\omega^2}{\exp(\frac{\gamma\omega(1-\beta\cos\theta)}{\mathrm{T}}) - 1} \,, \quad (5)$$

Integrating the number density over ω and $\cos\theta$, we get a dissociation time of the form $\tau_{dis} \approx \frac{4\alpha_s m_Q^2}{3\pi B}$,

$$B = \sum_{k=1}^{\infty} \left\{ f(v,T) \left(\frac{1}{\beta}\right) \sinh(\frac{k\gamma\beta\omega}{T}) \right\} e^{-\frac{k\gamma\omega_{\text{min}}}{T}} + \sum_{k=1}^{\infty} \left\{ g(v,T) \cosh(\frac{k\gamma\beta\omega}{T}) \right\} e^{-\frac{k\gamma\omega_{\text{min}}}{T}} \,. \quad (6)$$

where

$$f(v,T) = \left\{ \left(\frac{T}{k}\right)^2 \omega_{\text{min}} + \gamma\left(1 + \beta^2\right) \left(\frac{T}{k}\right)^3 \right\} \quad (7)$$

and

$$g(v,T) = \left\{ \left(\frac{T}{k}\right)^2 \omega_{\text{min}} + 2\gamma \left(\frac{T}{k}\right)^3 \right\} \quad (8)$$

The latter equation of the dissociation time indicates that the quarkonium breaks down faster when its velocity increases in the quark-gluon plasma–we have to be careful here because this might not happen in the case of screening. Although the velocity of the quarkonium is small, its dissociation time with respect to its velocity should not be ignored. For example, at a quark-gluon plasma temperature of 1 GeV, a non-moving quarkonium dissociates in a time duration of 3 fm/c, but if its velocity reaches 0.5 c, for instance, its dissociation time will decrease to 2.64

fm/c. If a quarkonium can reach a velocity of 0.9 c in 1 GeV quark-gluon plasma, which is unlikely to happen, because the transversal momentum, $P_T \sim v\gamma m_{\rm meson}$ can only be in the range of 5 GeV [11] due to beam constrains, the quarkonium dissociates in a time duration of 1.48 fm/c. It is easy to check that in the limit $v \to 0$, the dissociation time of Eq.(6) reduces to that of Eq.(4).

In conclusion, the quarkonium binding energy weakens with the increase of the temperature of the quark-gluon plasma and with the increase of the velocity of the quarkonium. But the velocity effect is smaller compared to that of the temperature. Thermalized gluons can play a major role in dissociating quarkonia, but since hard gluons can also be produced in quark-gluon plasma [12], dissociation of quarkonia due to collision with hard gluons needs to be investigated. This is a tricky problem though because we have to take into consideration the expansion of the plasma, and we cannot make use of the temperature because hard gluons escape the quark-gluon plasma before thermalization.

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