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Explicit derivation of a Central extended hyper-Kahler Metric.

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Abstract

This work consists in applying the analysis of integrable models to study the problem of hyper-Kahler metrics building. In this context, we use the harmonic superspace language applied to D=2 N=4 SU(2) Liouville self interacting model and derive an explicit central extended hyper-Kahler metric as well as the induced scalar potential.

I. INTRODUCTION

hyper-Kahler metrics building program is an important question of hyper-Kahler geometry in two and four dimensions and is solved in a nice way in the harmonic superspace [1-3]. More recently it was found that moduli spaces arising in topological field theories and other areas of mathematical physics possess hyper-Kahler structures, a notable example being the moduli space of BPS magnetic . monopoles [4].

Similar structures are also encountered in superstring theory, and in many other moduli problems of physical interest in quantum gravity [5-7]. In spite of these achievements, many things remain to be done. The classification of all complete, regular, hyper-Kahler manifolds remains an open question to this date, and even for some known examples, the explicit form of the metric has been difficult or impossible to determine so far. Recall that there are a few examples scattered in the literature that have been solved exactly.

Two of these examples are provided by the Eguchi-Hanson and the Taub-Nut metrics exhibiting both $U(2) = SU(2) \times U(1)$ isometries. Another example which is derived explicitly is the hyper-Kahler metric associated to the harmonic superspace Toda (Liouville) -like self- interaction and which can be useful in the sense that the particular hyper-Kahler geometrical structure that it is expected to describe, is connected to integrable models via the Toda-like self- interaction [3,8,9]. The explicit construction of a new series of metrics will certainly require a better and more systematic understanding of the unusual features that arise in the continual limit of the Toda field equations. In the present work, we compute explicitly the metric associated to D=2 N=4 SU(2) Liouville hyper-Kahler action in the harmonic superspace in the presence of central charges. We review in section 2 some general properties of the hyper-Kahler metric building program. Focusing to obtain the central extended bosonic action, we develop in section 3 some integrability techniques to derive the auxiliary fields F^{--}, G^{--} and the Lagrange field Δ, and present a method for their exact solvability. In section 4 we give the details concerning the derivation of the central extended metric as well as the induced scalar potential. The section 5 is devoted to our conclusion and discussion.

II. HYPER-KAHLER METRICS BUILDING PROGRAM IN 2D N=4 HS

We recall in this section some general results of the hyper-Kahler metric building from harmonic superspace. The subject of hyper-Kahler metrics building is an interesting problem of hyper-Kahler geometry that can be solved in a nice way in harmonic superspace if one knows how to solve the following non-linear differential equations on the sphere S^2[2].

$$\partial^{++}q^+ - \partial^{++}\left[\frac{\partial V^{4+}}{\partial\left(\partial^{++}\overline{q}^+\right)}\right] + \frac{\partial V^{4+}}{\partial \overline{q}^+} = 0$$
$$\partial^{++}q^+ + \partial^{++}\left[\frac{\partial V^{4+}}{\partial\left(\partial^{++}q^+\right)}\right] - \frac{\partial V^{4+}}{\partial q^+} = 0, \qquad (2.1)$$

where $q^+ = q^+(z,\overline{z},u^\pm)$ and its conjugates $\overline{q}^+ = \overline{q}^+(z,\overline{z},u^\pm)$ globally defined on $C \times S^2\left(S^2 \approx SU(2)\times U(1)\right)$ are complex fields parameterized by the local complex coordinates $(z,\overline{z})$ and the harmonic variables $u^\pm$ with . The symbol $\partial^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}$ stands for the so-called harmonic derivative and $V^{4+} = V^{4+}(q,u)$ is an interacting potential depending in general on $q^+, \overline{q}^+$; their derivatives and on the $u^\pm$'s . Note by the way that $q^+, \overline{q}^+$ may be expanded into an infinite series in powers of harmonic variables (for the bosonic part) preserving then the total U(1) charge in each term of the expansion as shown here below:

$$q^+(z,\overline{z},u) = u_i^+ f^i(z,\overline{z}) + u_i^+u_j^+u_k^- f^{(ijk)}(z,\overline{z}) + ... \qquad (2.2)$$

Note also that (2.1), which fixes the u-dependence of the q^+ 's , is in fact the pure bosonic projection of a two- dimensional N=4 supersymmetric HS superfield equation of motion. The remaining equations carry the spinor contributions and are shown to describe, among others, the space-time dynamics of the physical degrees of freedom namely $f^i(z,\overline{z})$, $\overline{f}^i(z,\overline{z})$, i=1,2 and their D=2, N=4 supersymmetric partners. By using the How-Stelle-Townsend (HST) realization of D=2, N=4 hypermultiplet $\left(O^4, \left(\frac{1}{2}\right)^4\right)$[10], Eqs.(2.1) read as

$$\partial^{++^2}\omega - \partial^{++}\left[\frac{\partial H^{4+}}{\partial(\partial^{++}\omega)}\right] + \frac{\partial H^{4+}}{\partial \omega} = 0 \qquad (2.3)$$

where $\omega = \omega(z,\bar{z},u)$ is a real field with zero U(1) charge defined on $C\times S^2$ and whose leading terms of its harmonic expansion given by

$$\omega(z,\overline{z},u) = u_i^+u_j^- f^{ij}(z,\overline{z}) + u_i^+u_j^+u_k^-u_l^- g^{ijkl}(z,\overline{z}) + ... \qquad (2.4)$$

Similar as in (2.1), the interacting potential H^{4+} depends in general on ω , its derivatives and the harmonics. Note the important observation of [11], that one can always pass from the q^+ hypermultiplet to the ω hypermultiplet via a duality transformation [12] by making a change of variables. In the remarkable case where the potentials H^{4+} and V^{4+} do not depend on the derivatives of the fields q^+ and ω (2.1,3) reduce then to

$$\partial^{++}q^+ + \frac{\partial V^{4+}}{\partial \overline{q}^+} = 0, \qquad (2.5)$$
$$\partial^{++^2}\omega + \frac{\partial H^{4+}}{\partial \omega} = 0. \qquad (2.6)$$

Since the solutions of these equations depend naturally on the potentials V^{4+} and H^{4+}, finding these solutions is not an easy task. There are only few examples scattered in the literature that have been solved exactly [2,9,13]. Let us review in what follows some them. The first example is the Taub-Nut (T.N) metric of D=4 Euclidean gravity. Its Potential V^{4+} reads as [2]

$$V^{4+}\left(q^+,\overline{q}^+\right) = \frac{\lambda}{2}\left(q^+\overline{q}^+\right)^2, \qquad (2.7)$$

where λ is a real coupling constant. According to this potential, the equation of motion is

$$\partial^{++}q^+ + \lambda q^+\left(\overline{q}^+\right)q^+ = 0 \qquad (2.8)$$

whose solution reads as

$$q^+(z,\bar{z},u) = u_i^+ f^i(z,\bar{z}) \exp(-\lambda u_k^+u_l^- f^{(k}f^{l)}). \quad (2.9)$$

Note that the knowledge of this solution is an important step towards identifying the metric of the manifold parameterized by $f^i(z,\overline{z})$ and $\overline{f}^i(z,\overline{z})$ of the D=2 N=4 supersymmetric non linear Taub-Nut σ - model. The latter possesses an action whose bosonic part reads as

$$S_B^{TN} = -\frac{1}{2}\int dzd\overline{z}g_{ij}\partial_z f^i\partial_{\overline{z}}f^j - \frac{1}{2}\int dzd\overline{z}\left(\overline{g}^{ij}\partial_z\overline{f}_i\partial_{\overline{z}}\overline{f}_j + 2h_j^i\partial_z f^j\partial_{\overline{z}}\overline{f}_i\right) \quad (2.10)$$

and the metric is known to be[14]

$$g_{ij} = \frac{\lambda\left(2+\lambda f\overline{f}\right)}{2\left(1+\lambda f\overline{f}\right)}\overline{f}_i\overline{f}_j,$$
$$\overline{g}^{ij} = \frac{\lambda\left(2+\lambda f\overline{f}\right)}{2\left(1+\lambda f\overline{f}\right)}f_if_j, \qquad (2.11)$$
$$h_j^i = \delta_j^i\left(1+\lambda f\overline{f}\right) - \frac{\lambda\left(2+\lambda f\overline{f}\right)}{2\left(1+\lambda f\overline{f}\right)}f_i\overline{f}_j,$$
$$f\overline{f} \equiv f^i\overline{f}_i.$$

In the standard Taub-Nut form [15], we have

$$ds^2 = \frac{r+M}{2(r-M)}dr^2 + \frac{1}{2}\left(r^2 - M^2\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) + 2M^2\left(\frac{r-M}{r+M}\right)\left(d\psi + \cos\theta d\varphi\right)^2 \quad (2.12)$$

once the following change of variables [14]

$$\begin{aligned} f^1 &= \sqrt{2M(r-M)}\cos\tfrac{\theta}{2}\exp\tfrac{i}{2}(\psi+\varphi), \\ f^2 &= \sqrt{2M(r-M)}\sin\tfrac{\theta}{2}\exp\tfrac{i}{2}(\psi-\varphi), \end{aligned} \quad (2.13)$$

is performed with $f\overline{f} = 2M(r-M)$, $r > M \equiv \frac{1}{2\sqrt{\lambda}}$.

The second example we consider is the Eguchi-Hanson (E.H) model. This has also been solved exactly and corresponds to the following potential:

$$H^{4+}(\omega) = \left[u_i^+ u_j^+ \xi^{(ij)}\right]^2 \times \omega^2 \quad (2.14)$$

where is an SU(2) real constant triplet. Thus, unlike the T.N action, the E.H one contains explicit harmonics. More details are exposed in [2].

Recently, a new integrable model has been proposed [9]. This model was obtained by focusing on (2.5b) and looking for potentials leading to exact solutions of this equation. The method used in this issue consist in suggesting new plausible integrable equations by proceeding in formal analogy with the known integrable two dimensional non-linear differential equations, especially the Liouville equation and its Toda generalizations [16].

The important result in this sense was the proposition of the following potential:

$$H^{4+}(\omega, u) = \left(\frac{\xi^{++}}{\lambda}\right)^2 \exp(2\lambda\omega), \quad (2.15)$$

which leads, to the following non linear differential equation of motion:

$$\lambda\left(\partial^{++}\right)^2\omega - \xi^{++^2}e^{2\lambda\omega} = 0. \quad (2.16)$$

Using the formal analogy with the SU(2) Liouville equation, we showed that (2.15) is integrable. The explicit solution of this non-linear differential equation is [9]

$$\xi^{++}e^{\lambda\omega} = \frac{u_i^+ u_j^+ f^{ij}(z,\overline{z})}{1 - u_k^+ u_l^- f^{kl}(z,\overline{z})}. \quad (2.17)$$

Furthermore, the origin of the integrability in (2.15) is shown to deal with the existence of a symmetry (conformal symmetry) generated by the following conserved current

$$T^{4+} = \partial^{++2} - \frac{1}{\lambda}\partial^{++2}\omega, \quad (2.18)$$

with $\partial^{++}T^{4+} = 0$. Note that other examples of hyper-Kahler potentials are known in the literature [13].

III. CENTRAL EXTENSION OF THE BOSONIC ACTION AND INTEGRABILITY

Focusing in this section on the above SU(2) Liouville model, to derive the corresponding extended hyper-Kahler metric associated to the proposed potential (2.14) in the presence of central charges. The general procedure to get the component form of the bosonic non-linear sigma-model using the harmonic superspace approach consists in applying the method presented in [2].One starts by writing the action describing the general coupling of the analytic superfield Ω

we are interested in and deriving the corresponding equation of motion. For this action, corresponding to some hyper-Kahler manifold, one only has to expand the equations of motion in Grassman variables θ and ignore all the fermionic field components. Then one has to solve the kinematical differential equations on the sphere S^2 for the auxiliary field components a fact which leads to eliminate the infinite tower of them in the harmonic expansion of the hypermultiplet HSS superfields. Substituting the solution into the original action in HSS and integrating over all the harmonic variables and the anti-commuting coordinates yields the required component form of the action form which the hyper-Kahler metric can be extracted.

Let us first recall that the harmonic superspace (HSS) is parameterized by the super-coordinates $Z^M = \left(Z_A^M, \theta_r^-, \overline{\theta}_r^-\right)$, where $Z_A^M = \left(Z, \overline{Z}, \theta_r^+, \overline{\theta}_r^+, u^\pm\right)$ are the super coordinates of the so-called analytic subspace in which D=2 N=4 supersymmetric theories are formulated. The integral measure of the harmonic superspace is given in the Z_A^M basis by $d^2Zd^4\theta^+du$. The matter superfield $\left(O^4(1/2)^4\right)$ is realized by two dual analytic superfields $Q^+ = Q^+\left(Z, \overline{Z}, \theta^+, \overline{\theta}^+, u\right)$ and $\Omega = \Omega(z, \overline{z}, \theta^+, \overline{\theta}^+, u)$, whose leading bosonic fields are respectively given by q+ and ω Eqs(2.2, 4). The model we are interested in and which describes the coupling of the analytic superfield Ω, is given by the following action [9]

$$S[\Omega] = \int d^2zd^4\theta^+du\frac{1}{2}\left(D^{++}\Omega\right)^2$$

$$\int d^2zd^4\theta^+du + \frac{1}{2}\left(\frac{\xi^{++}}{\lambda}\right)^2 e^{2\lambda\Omega} \quad (3.1)$$

where λ is the coupling constant of the model and $\xi^{++} = u_i^+u_j^+\xi^{ij}$ a constant isotriplet similar to that appearing in the E.H model and whereD^{++} is the harmonic derivative whose central extension is [17]

$$D^{++} = \partial^{++} - 2\overline{\theta}^+_+\theta^+_+\partial_{--} - 2\overline{\theta}^+_-\theta^+_-\partial_{++} + \theta^+_+\theta^+_-\overline{Z} - \overline{\theta}^+_-\overline{\theta}^+_+Z \quad (3.2)$$

where the central charges $(Z, \bar{Z})$ are generators which belongs to the Cartan subgroup of a given Lie group. The HSS equation of motion for the analytic superfield Ω reads:

$$\lambda D^{++^2}\Omega - \xi^{++^2}e^{2\lambda\Omega} = 0 \quad (3.3)$$

where Ω can by expanded in a series of θ_r^+ and $\bar{\theta}_r^+, r = \pm$ as

$$\begin{aligned}\Omega = \omega &+ \left[\theta^+_-\theta^+_+F^{--} + \bar{\theta}^+_+\bar{\theta}^+_-\bar{F}^{--}\right] \\ &+ \left[\bar{\theta}^+_-\theta^+_+G^{--} + \bar{\theta}^+_+\theta^+_-\bar{G}^{--}\right] \\ &+ \left[\bar{\theta}^+_-\theta^+_-B^{--}_{++} + \bar{\theta}^+_+\theta^+_+B^{--}_{--}\right] \\ &+ \left[\bar{\theta}^+_-\theta^+_-\bar{\theta}^+_+\theta^+_+\Delta^{-4}\right]\end{aligned} \quad (3.4)$$

In the presence of central charges, some of the kinematical equations of motion in the (Z_A, u) space get modified

$$0 = \lambda\partial^{++2}\omega - \xi^{++2}e^{2\lambda\omega} = 0, \quad (3.5)$$

$$0 = \partial^{++2}F^{--} - 2\xi^{++2}F^{--}e^{2\lambda\omega} - 2\bar{Z}\partial^{++}\omega, \quad (3.6)$$

$$0 = \partial^{++2}\bar{F}^{--} - 2\xi^{++2}\bar{F}^{--}e^{2\lambda\omega} + 2Z\partial^{++}\omega, \quad (3.7)$$

$$0 = \partial^{++2}G^{--} - 2\xi^{++2}G^{--}e^{2\lambda\omega}, \quad (3.8)$$

$$0 = \partial^{++2}\bar{G}^{--} - 2\xi^{++2}\bar{G}^{--}e^{2\lambda\omega}, \quad (3.9)$$

$$0 = -4\partial^{++}\partial_{++}\omega\partial^{++2}B^{--}_{++} - 2\xi^{++2}B^{--}_{++}e^{2\lambda\omega}, \quad (3.10)$$

$$0 = -4\partial^{++}\partial_{--}\omega\partial^{++2}B^{--}_{--} - 2\xi^{++2}B^{--}_{--}e^{2\lambda\omega}, \quad (3.11)$$

$$\begin{aligned}0 = \partial^{++2}\Delta^{-4} &- 4\partial^{++}\partial_{--}B^{--}_{++} \\ &-4\partial^{++}\partial_{++}B^{--}_{--} + 8\partial_{--}\partial_{++}\omega \\ &-2\bar{Z}\partial^{++}\bar{F}^{--} + 2Z\partial^{++}F^{--} - 2\left|Z\right|^2\omega \\ &-2\,\xi^{++2}e^{2\lambda\omega}\times \\ &\left\{\Delta^{-4} + 2\lambda\left(F^{--}\bar{F}^{--} - G^{--}\bar{G}^{--} + B^{--}_{++}B^{--}_{--}\right)\right\}\end{aligned} \quad (3.12)$$

The Liouville like equation of motion (3.5) is a constraint equation fixing the dependence of ω in terms of the physical bosonic fields $f^{(ij)}$ of the D=2 N=4 hypermultiplet. As it is already discussed in [3,9]; the Knowledge of the explicit solution of this nonlinear differential equation is a crucial step in this program. The second set of relations (3.6-8) describes the equations of motion of the auxiliary fields F^{--} and G^{--} and of canonical dimension one. (3.9) gives the equation of motion of the Lagrange field of canonical dimension two in terms of and the other auxiliary fields. Note that the principal contributions of the central charges appear only on the auxiliary fields F^{--}, $\overline{F}^{--}$ and the Lagrange field Δ^{-4}. Furthermore, the fields F^{--} and G^{--} are shown to behave in a similar way in the absence of central charges. To solve these equations of motion, one starts first by solving the Liouville-like equation of motion (3.5) whose solution [16]; originated from integrability and conformal symmetry in two dimensions is given in the HS language by [9]

$$\lambda\partial^{++}\omega = \xi^{++}e^{\lambda\omega} = \frac{f^{++}}{1-f}\ , \quad (3.13)$$

with

$$f = u^+_{(i}u^-_{j)}f^{(ij)} + f^0$$
$$f^{++} = u^+_{(i}u^+_{j)}f^{(ij)} \quad (3.14)$$

After integrating over the Grassmann variables in the action (3.1) one finds that the bosonic action reduces to

$$\begin{aligned}S = &\int d^2zdu\partial^{++}\omega\partial^{++}\Delta^{-4} \\ &+\frac{1}{\lambda}\xi^{++^2}\int d^2zdue^{2\lambda\omega}\Delta^{-4} \\ &+\int d^2zdu\partial^{++}F^{--}\partial^{++}\overline{F}^{--} \\ &+2\xi^{++^2}\int d^2zdue^{2\lambda\omega}F^{--}\overline{F}^{--} \\ &-\int d^2zdu\partial^{++}G^{--}\partial^{++}\overline{G}^{--} \\ &+2\xi^{++^2}\int d^2zdue^{2\lambda\omega}G^{--}\overline{G}^{--} \\ &+\int d^2zdu\partial^{++}B^{--}_{--}\partial^{++}B^{--}_{++} \\ &-2\int d^2zdu\partial^{++}\omega\left(\partial_{--}B^{--}_{++} + \partial_{++}B^{--}_{--}\right) \\ &+4\int d^2zdu\partial_{--}\omega\partial_{++}\omega \\ &+2\xi^{++^2}\int d^2zdue^{2\lambda\omega}B^{--}_{++}B^{--}_{--} \\ &-2\int d^2zdu\partial_{++}\omega\partial^{++}B^{--}_{--} \\ &+\int d^2zdu\partial_{--}\omega\partial^{++}B^{--}_{++} \\ &-\int d^2zdu\overline{Z}\left(\partial^{++}\omega\overline{F}^{--} + \partial^{++}\overline{F}^{--}\omega\right)\end{aligned} \quad (3.15)$$

$$
\begin{aligned}
&-\int d^2zdu\,|Z|^2\,\omega^2\\
&+\int d^2zduZ\left(\partial^{++}\omega F^{--}+\partial^{++}F^{--}\omega\right)
\end{aligned}
$$

Using the equation of motion for ω (3.5), one shows that the following harmonic integrals vanish

$$
\begin{aligned}
0 &= \int du\partial^{++}\omega\partial^{++}\Delta^{-4}\\
&+\int du\frac{1}{\lambda}\xi^{++^2}e^{2\lambda\omega}\Delta^{-4}, \qquad (3.16)\\
0 &= \int du\partial^{++}G^{--}\partial^{++}\overline{G}^{--}\\
&+2\xi^{++^2}\int due^{2\lambda\omega}G^{--}\overline{G}^{--}, \qquad (3.17)\\
0 &= \int du\overline{Z}\partial^{++}\omega\overline{F}^{--}+\partial^{++}\overline{F}^{--}\omega, \qquad (3.18)\\
0 &= \int duZ\left(\partial^{++}\omega F^{--}+\partial^{++}F^{--}\omega\right). \qquad (3.19)
\end{aligned}
$$

These vanishing integrals serve to eliminate the auxiliary fields G and Δ . The resulting bosonic action is then

$$
\begin{aligned}
S = 4&\int d^2zdu\partial_{--}\omega\partial_{++}\omega\\
-2&\int d^2zdu\partial^{++}\omega\partial_{--}B^{--}_{++}\\
-2&\int d^2zdu\partial^{++}\omega\left(\partial_{++}B^{--}_{--}+\overline{ZF}^{--}\right)\\
+&\int d^2zdu\left(2\partial_{++}\omega\partial^{++}B^{--}_{--}-|Z|^2\,\omega^2\right)\\
-&\int d^2zdu\left(2\partial_{--}\omega\partial^{++}B^{--}_{++}\right) \qquad (3.20)
\end{aligned}
$$

Note that at $\left(Z,\overline{Z}\right)=0$, the auxiliary field F now contribute too unlike that in ref[3]. To obtain a purely bosonic theory; one have to reduce much more this action. To do this; one needs to solve the non linear differential equation for the auxiliary fields F and B. Using some algebraic manipulations based on the knowledge of the Liouville-like solution (3.10) and requiring the consistency we propose the following solutions for F and B (3.6,8)

$$
\begin{aligned}
F^{--} &= \frac{1}{\xi^{++}}\left\{-\frac{\overline{Z}}{\lambda}e^{-\lambda\omega}+\overline{\alpha}e^{\lambda\omega}\right\},\\
\overline{F}^{--} &= \frac{1}{\xi^{++}}\left\{+\frac{Z}{\lambda}e^{-\lambda\omega}-\alpha e^{\lambda\omega}\right\} \qquad (3.21)
\end{aligned}
$$

and

$$B^{--}_{rr}=\frac{1}{\xi^{++}}\left\{-2\partial_{rr}\omega\,e^{-\lambda\omega}+\eta_{rr}\,e^{\lambda\omega}\right\},\ r=\pm \qquad (3.22)$$

where α *and* η are two arbitrary constants which satisfy $\bar{\alpha}=-\alpha$ and $\partial^{++}\eta_{rr}=0$. (3.18) are obtained by proceeding by steps. The first step consists in finding an homogeneous solution (for $Z=\bar{Z}=0$) and then propose a particular solution leading then to the general form (3.18). The explicit solution for B^{--}_{rr} contains also an homogeneous solution ($\partial^{++}\partial_{--}\omega=0$)

$$B^{--}_{rr}=\frac{\eta_{rr}}{\xi^{++}}e^{\lambda\omega}, \qquad (3.23)$$

and a particular one

$$B^{--}_{rr}=\frac{-2}{\xi^{++}}\partial_{rr}\omega\,e^{-\lambda\omega} \qquad (3.24)$$

Its worth pointing out that the particular solution for F and B are simply obtained thanks to the Liouville (3.5) just by assuming that $\partial^{++^2}F^{--}=0$ and $\partial^{++^2}B^{--}_{rr}=0$. Furthermore note that another particular solution of B^{--}_{rr} was proposed in [3] namely

$$B^{--}_{rr}=\xi^{--}\partial_{rr}\omega \qquad (3.25)$$

where $\xi^{--}=u^-_{(i}u^-_{j)}\xi^{(ij)}$ is an SU(2) triplet constant required to satisfy $\partial^{++}\xi^{--}=2$. Although they appear to be different, the two particular solutions (3.21, 3.22) share naturally a crucial property namely

$$\partial^{++}\xi^{--}=\partial^{++}(-2\frac{e^{-\lambda\omega}}{\xi^{++}})=2. \qquad (3.26)$$

On the other hand; setting $Z=\bar{Z}=0$ in (3.6), one can derive a solution for the auxiliary fields $G^{--}\,and\,\bar{G}^{--}$ induced from the striking resemblance between the equations for F and G. One have

$$
\begin{aligned}
G^{--} &= \frac{\bar{\alpha}}{\xi^{++}}e^{\lambda\omega},\\
\bar{G}^{--} &= -\frac{\alpha}{\xi^{++}}e^{\lambda\omega} \qquad (3.27)
\end{aligned}
$$

Using the previous results; note also that when $Z=\bar{Z}=0$; the Lagrange field Δ^{-4} is shown to satisfy

$$
\begin{aligned}
&\partial^{++2}\Delta^{-4}+8\partial_{--}\partial_{++}\omega\\
&-4e^{2\lambda\omega}\left[4\lambda(\eta_{++}\partial_{--}\omega+\eta_{--}\partial_{++}\omega)\right]\\
&-4e^{2\lambda\omega}\left[\partial_{--}\eta_{++}+\partial_{++}\eta_{--}\right]\\
&=2\xi^{++2}\Delta^{-4}e^{2\lambda\omega} \qquad (3.28)\\
&+4\lambda(\eta_{++}\eta_{--}e^{4\lambda\omega}+4\partial_{++}\omega\partial_{--}\omega).
\end{aligned}
$$

whose solution ; upon setting for simplicity $\eta_{rr}=0$; is

$$
\begin{aligned}
\Delta^{-4} &= \frac{1}{\xi^{++2}}\left(\bar{\alpha}e^{\lambda\omega}+4e^{-2\lambda\omega}\partial_{--}\partial_{++}\omega\right)\\
&\frac{-8e^{-2\lambda\omega}}{\xi^{++2}}\left(\lambda\partial_{++}\omega\partial_{--}\omega\right). \qquad (3.29)
\end{aligned}
$$

To summarize, we have presented in this section the integrability mechanism applied to the problem of hyper-Kahler metrics building in the case of the central extension of D=2 N=4 SU(2) Liouville self interaction. To accomplish this program we have showed that the resulting system of non linear differential equations is integrable and derived the explicit solutions for the auxiliary fields F, B, G and Δ . Note that only F and B which are needed in computing the pure bosonic action (3.17). The latter is shown to correspond to the following form

$$\begin{aligned} S \ &= \ -4\int d^2zdu\partial_{--}\omega\partial_{++}\omega \\ &+\tfrac{8}{\lambda}\int d^2zdu\partial_{--}\partial_{++}\omega + e^{2\lambda\omega} \\ &-\tfrac{2}{\lambda}\int d^2zdu\left(\left(\partial_{--}\eta_{++}+\partial_{++}\eta_{--}-\alpha\overline{Z}\right)\right) \\ &-4\int d^2zdu\left(\eta_{++}\partial_{--}\omega\right) \\ &-\int d^2zdu\left|Z\right|^2\left(\omega^2+\frac{2}{\lambda^2}\right) \end{aligned} \quad (3.30)$$

giving then the dependence of the action only on the bosonic degrees of freedom $f = u_i^+u_j^-f^{ij}$. What remains to be done now is to use the equation of motion (3.10) for ω and integrate over the harmonic variables $u^{\pm}$ to derive the purely bosonic action, from which one can easily identify the metric associated to the D=2 N=4 SU(2) Liouville self interaction.

IV. 4-BUILDING THE EXTENDED HYPER-KAHLER METRIC

Starting from the action (3.27) and using the convenient parameterization [9]

$$\xi^{++}e^{\lambda\omega} = \frac{f^{++}}{1-f} \quad (4.1)$$

we have

$$\partial_{rr}\omega = \frac{1}{\lambda}\left[\frac{\partial_{rr}f^{++}}{f^{++}} + \frac{\partial_{rr}f}{1-f}\right]. \quad (4.2)$$

Furthermore one way to write , which appears in the last term of the action (3.27); in terms of the bosonic degrees of freedom f, is to consider for simplicity the following "approximation"

$$\omega^2 \approx \frac{e^{\lambda\omega}+e^{-\lambda\omega}-2}{\lambda^2} \quad (4.3)$$

which leads to set

$$\omega^2 \approx \frac{1}{\lambda^2}\left[\frac{f^{++}}{\xi^{++}(1-f)} - 2\right] \quad (4.4)$$

for f around $f = u^{+i}u_i^-$. Inserting these expressions (4.2, 4) into the bosonic action (3.27), one obtains the following result

$$\begin{aligned} S = &\int d^2zdu\frac{-12}{\lambda^2}\left[\frac{\partial_{++}f^{++}\partial_{--}f^{++}}{f^{++^2}}\right] \\ &\int d^2zdu\frac{8}{\lambda^2}\left[\frac{\partial_{--}\partial_{++}f^{++}}{f^{++}}+\frac{\partial_{--}\partial_{++}f}{1-f}\right] \\ &\int d^2zdu\frac{-4}{\lambda^2}\left[\frac{\partial_{++}f\partial_{--}f^{++}}{f^{++}(1-f)}+\frac{\partial_{++}f^{++}\partial_{--}f}{(1-f)f^{++}}\right] \\ &\int d^2zdu\frac{4}{\lambda^2}\left[\frac{\partial_{++}f\partial_{--}f}{(1-f)^2}\right] \\ &\int d^2zdu\frac{-4\eta_{++}}{\lambda}\left[\frac{f^{++}\partial_{--}f}{\xi^{++^2}(1-f)^2}+\frac{f^{++^2}\partial_{--}f}{\xi^{++^2}(1-f)^3}\right] \\ &\int d^2zdu\frac{-\left|Z\right|^2}{\lambda^2}\left[\frac{f^{++}}{\xi^{++}(1-f)}\right] \\ &\int d^2zdu\frac{-2}{\lambda}\left(\partial_{--}\eta_{++}+\partial_{++}\eta_{--}-\alpha\overline{Z}\right) \\ &\times\left[\frac{f^{++^2}}{\xi^{++^2}(1-f)^2}\right] \end{aligned} \quad (4.5)$$

This action contains terms with a singularity around $f = u^{+i}u_i^-$. We point out that this singularity is shown to originate from the solution of the Toda - (Liouville)- like equation of motion (3.10). As it is worth stressing that not every solution of the continual Toda-(Liouville)- equation of motion has a good space-time interpretation, since the corresponding metric might be incomplete with singularities, it seems at first sight that our hyper-Kahler metric will be incomplete. But using some algebraic manipulations, one can derive the complete form of the metric inspired from the first leading terms in a nice way.

To derive the metric from the bosonic action (4.5) , one carries out the following operations step-by-step. Considering first the following approximation:

$$\frac{1}{(1-f)^{\epsilon}} = \sum_{i=0}^{\infty}\frac{(\varepsilon+i-1)!}{(\epsilon-i)!i!}\ ,\ \varepsilon = 1,2,3,... \quad (4.6)$$

with $f = u^+_{(i}u^-_{j)}f^{(ij)} + f^0$. Next, substitute this expression into (4.5) and integrate over the harmonics once the power $f^i\left(Z,\overline{Z}\right)$ of the bosonic field f in (4.6) are expressed as series in terms of $f^{(ij)}$, f^0 and the symmetrized product of harmonics. To do this one also has to use the standard reduction identities [2]

$$\begin{aligned} 0 = &-u_i^+u^+_{(j_1...}u^+_{j_n}u^-_{k_1...}u^-_{k_m)} + u^+_{(i}u^+_{j_1...}u^+_{j_n}u^-_{k_1...}u^-_{k_m)} \\ &+\frac{m}{m+n+1}\varepsilon_{i(k_1}u^+_{j_1...}u^+_{j_n}u^-_{k_1...}u^-_{k_m)} \end{aligned} \quad (4.7)$$

$$\begin{aligned} 0 = &-u_i^-u^+_{(j_1...}u^+_{j_n}u^-_{k_1...}u^-_{k_m)} + u^+_{(i}u^+_{j_1...}u^+_{j_n}u^-_{k_1...}u^-_{k_m)} \\ &-\frac{m}{m+n+1}\varepsilon_{i(j_1}u^+_{j_1...}u^+_{j_n}u^-_{k_1...}u^-_{k_m)} \end{aligned} \quad (4.8)$$

and the $u_i^{\pm}$ integration rules

$$\int du \left(u^+\right)^{(m} \left(u^-\right)^{n)} \left(u^+\right)_{(k} \left(u^-\right)_{l)} = \begin{cases} \frac{(-1)^n m!n!}{(m+n+1)!}\delta_{(j_1}^{(i_1}...\delta_{j_{k+l})}^{i_{m+n})} & if\ m=l,\ n=k \\ 0 & otherwise \end{cases} \quad (4.9)$$

with

$$\left(u^+\right)^{(m} \left(u^-\right)^{n)} \equiv u^{+(i_1}...u^{+i_m}u^{-j_1}...u^{-j_n)} \quad (4.10)$$

Lengthy and very hard calculations lead finally to the following purely bosonic action:

$$\begin{aligned} S[f] = & \int d^2z A_{ijkl}\partial_{++}f^{(ij)}\partial_{--}f^{(kl)} \\ & +\int d^2z C_{ij}\partial_{++}\partial_{--}f^{(ij)} \\ & +\int d^2z B_{ij}\partial_{++}f^{(ij)}\partial_{--}f^0 \\ & +\int d^2z B_{ij}\partial_{++}f^0\partial_{--}f^{(ij)} \\ & +\int d^2z \left(D\partial_{++}\partial_{--}f^0 + E\partial_{++}f^0\partial_{--}f^0\right) \\ & +\int d^2z \left(F\partial_{++}f^{(ij)}\partial_{--}f_{(ij)} + V(f)\right) \end{aligned} \quad (4.11)$$

where

$$V(f) = G_{++}\partial_{--}f^0 + K_{++(ij)}\partial_{--}f^{(ij)} + L. \quad (4.12)$$

From (4.10) one can easily derive the metric. The tensor components of this metric are A_{ijkl}, B_{ij}, C_{ij}, D, E and F, such that

$$\begin{aligned} A_{ijkl} &= A_{jikl} = A_{ijlk} \\ B_{ij} &= B_{ji} \\ C_{ij} &= C_{ji} \end{aligned} \quad (4.13)$$

The first explicit expression obtained for this metric is of course incomplete due to the previous approximation. The missing terms in this metric as well as (the potential V(f)) are easily recuperated by looking just at the behavior of the first leading terms of the components A_{ijkl}, B_{ij}, C_{ij}, D, E, F and $\left(G_{++},\ K_{++(ij)},\ L\right)$. The complete expression of the metric described by the bosonic field f reads as:

$$\begin{aligned} A_{ijkl} = & -\frac{24}{\lambda^2}\frac{1}{f^{(ij)}f^{(kl)}} \\ & +\frac{2}{3\lambda^2}\sum_{N=0}\left(\frac{f_{(ij)}}{f_{(kl)}}+\frac{f_{(kl)}}{f_{(ij)}}\right)\times \\ & \left\{C_{N+1}^N f^{0N} + \sum_{n=1}A(N,n)\, f^{0N-2n}(ff)^n\right\} \end{aligned} \quad (4.14)$$

$$\begin{aligned} B_{ij} = & -\frac{4}{3\lambda^2}\sum_{N=0}f_{(ij)}2C_{N+2}^N f^{0N} \\ & -\frac{4}{3\lambda^2}\sum_{N=0}f_{(ij)}\sum_{n=1}B_1(N,n)\, f^{0N-2n}(ff)^n \\ & -\frac{8}{\lambda^2}\sum_{N=0}f^{0N}\frac{1}{f^{(ij)}} \\ & -\frac{8}{\lambda^2}\sum_{N=0}\sum_{n=1}B_2(N,n)\, f^{0N-2n}(ff)^n\frac{1}{f^{(ij)}}, \end{aligned} \quad (4.15)$$

$$\begin{aligned} C_{ij} = & \frac{8}{\lambda^2}\frac{1}{f^{(ij)}} - \frac{4}{3\lambda^2}\sum_{N=0}C_{N+1}^N f^{0N} \\ & -\frac{4}{3\lambda^2}\sum_{N=0}\sum_{n=1}C(N,n)\, f^{0N-2n}(ff)^n f_{(ij)} \end{aligned} \quad (4.16)$$

$$\begin{aligned} D = & \frac{8}{\lambda^2}\sum_{N=0}f^{0N} \\ & +\frac{8}{\lambda^2}\sum_{N=0}\sum_{n=1}D(N,n)\, f^{0N-2n}(ff)^n \end{aligned} \quad (4.17)$$

$$\begin{aligned} E = & \frac{4}{\lambda^2}\sum_{N=0}C_{N+1}^N f^{0N} \\ & +\frac{4}{\lambda^2}\sum_{N=0}\sum_{n=1}E(N,n)\, f^{0N-2n}(ff)^n \end{aligned} \quad (4.18)$$

$$\begin{aligned} F = & -\frac{2}{3\lambda^2}\sum_{N=0}C_{N+1}^N f^{0N} \\ & -\frac{2}{3\lambda^2}\sum_{N=0}\sum_{n=1}F(N,n)\, f^{0N-2n}(ff)^n \end{aligned} \quad (4.19)$$

where $(ff) = f^{(ij)}f_{(ij)}$. Furthermore the coefficients A(N,n), Bi(N,n),... , F(N,n) are finite numerical quantities defined for $n \geq 1$ and $N \geq n$. Some examples are

$$\begin{aligned} A_1(2,1) &= -\frac{1}{5},\ A_2(2,1) = -\frac{15}{14},... \\ B_1(2,1) &= -\frac{4}{5},\ B_2(2,1) = -\frac{1}{6},... \\ C(2,1) &= -\frac{1}{5},\ D(2,1) = -\frac{1}{6},... \\ E(2,1) &= -\frac{1}{2},\ F(2,1) = -\frac{1}{2},... \end{aligned} \quad (4.20)$$

Another interesting consequence of the presence of central charges is a non trivial scalar potential V(f) which is given by

$$\begin{aligned} V(f) = & -\frac{8}{\lambda}\eta_{++}\sum_{N=0}\frac{f^{(ij)}f^{(kl)}}{\xi^{(ij)}\xi^{(kl)}}\partial_{--}f^0 C_{N+2}^N f^{0N} \\ & -\frac{8}{\lambda}\eta_{++}\sum_{N=0}\frac{f^{(ij)}f^{(kl)}}{\xi^{(ij)}\xi^{(kl)}}\partial_{--}f^0\times \\ & \sum_{n=1}G(N,n)\, f^{0N-2n}(ff)^n \end{aligned}$$

$$+\frac{4}{3\lambda}\eta_{++}x\sum_{N=0}\frac{f^{(kl)}f^{(pq)}}{\xi^{(kl)}\xi^{(pq)}}\times$$
$$f_{(ij)}\partial_{--}f^{(ij)}3C^N_{N+3}f^{0N}$$
$$+\frac{4}{3\lambda}\eta_{++}x\sum_{N=0,n=1}\frac{f^{(kl)}f^{(pq)}}{\xi^{(kl)}\xi^{(pq)}}f_{(ij)}\partial_{--}f^{(ij)}\times$$
$$K(N,n)\,f^{0N-2n}\,(ff)^n$$
$$-\frac{4}{\lambda}\left(\partial_{--}\eta_{++}+\partial_{++}\eta_{--}-\alpha\overline{Z}\right)\times$$
$$\sum_{N=0}C^N_{N+1}f^{0N}\frac{f^{(ij)}f^{(kl)}}{\xi^{(ij)}\xi^{(kl)}}$$
$$\sum_{N=0,n=1}L_1(N,n)\,f^{0N-2n}\,(ff)^n\,\frac{f^{(ij)}f^{(kl)}}{\xi^{(ij)}\xi^{(kl)}}$$
$$-\frac{|Z|^2}{\lambda^2}\sum_{N=0}f^{0N}\frac{f^{(ij)}}{\xi^{(ij)}}$$
$$-\frac{|Z|^2}{\lambda^2}\sum_{N=0,n=1}L_2(N,n)\times \qquad (4.21)$$
$$f^{0N-2n}\,(ff)^n\,\frac{f^{(ij)}}{\xi^{(ij)}}$$

where α and η_{rr} are arbitrary constant already introduced in section 3. The coefficients G(N,n), K(N,n) and Li(N,n) are also finite numerical quantities defined for $n \geq 1$ and $N \geq 2n$. Particular examples are given by

$$\begin{aligned} G(2,1) &= -1,\ G(3,1)=-5,\\ K(2,1) &= -\frac{1}{3},\ K(3,1)=-2,\\ L_1(2,1) &= -\frac{1}{2},\ L_1(3,1)=-2,\\ L_2(2,1) &= -\frac{1}{6},\ L_2(3,1)=-\frac{1}{2},\ldots \end{aligned} \qquad (4.22)$$

Note that a possibility of generating non-trivial scalar potential via non-vanishing central charges in the non linear 2D, N=4 supersymmetric sigma-models was noticed earlier by Alvarez-Gaumé and Freedman in ref [5].

V. CONCLUSION AND DISCUSSION:

In the spirit to extend the results already established in [3]and which deal with an explicit derivation of a new hyper-Kahler metric associated to D=2 N=4 SU(2) Liouville self interacting model, we have tried to look for the contribution of central charges operators in the theory under consideration. This central extension is shown to give rise to a non trivial scalar potential $V(f)$ depending on the physical bosonic field f. The major content of this work can be summarized as follows:

1. Having shown how the Liouville-like equation of motion Eq(3.5) leads to the bosonic central extended action Eq(3.17) depending on $\omega, B^{--}_{\pm\pm}, \overline{F}^{--}$, our first principal task was to search to reduce much more this action, a fact which will help to extract easily the associated hyper-Kahler metric. To do this one have to solve the non linear differential equations for the auxiliary fields $B^{--}_{\pm\pm}, \overline{F}^{--}$. Typical solutions generalizing the ones obtained in [3] are proposed. The originality related to these equations consists in solving not only the non linear equations for the fields $B^{--}_{\pm\pm}$ but also for the field $\overline{F}^{--}$ (recall that the latter is integrated out when $Z=\overline{Z}=0$ as shown in [3]). The proposed explicit solutions are given in Eqs(3.18-19) and the resulting reduced bosonic action is shown to take the form presented in Eq(3.27).

2. In computing the metric, we have observe that the factor proportional to $|Z|^2$ in the action Eq(3.27) namely $\omega^2+\frac{2}{\lambda^2}$ is nothing but the first leading contribution of the non linear function $\frac{e^{\lambda\omega}+e^{-\lambda\omega}}{\lambda^2}$ for which $e^{\lambda\omega}$ is just the Liouville scalar potential satisfying Eq(3.10). This is an important step towards achieving the reduction procedure of the action to the purely bosonic form. One of the advantages of this observation is that one can overcome the difficulty in treating the term $\omega^2+\frac{2}{\lambda^2}$ by using simply Eq(4.1) which gives approximately a factor $\frac{|Z|^2}{\lambda^2}\left(\frac{f^{++}}{\xi^{++}(1-f)}+\frac{\xi^{++}(1-f)}{f^{++}}\right)$.

3. The use of the approximation Eq(4.6) is important in computing the singular term in Eq(4.5) once the standard reduction identities are used. The missing terms in doing the algebraic computations are naturally recuperated by looking just at the behaviour of the first leading ones. This gives once again the possibility of interpreting the resulting metric associated to the $\left(Z,\overline{Z}\right)$ extended D=2 N=4 SU(2) Liouville self interacting model as a complete hyper-Kahler metric.

4. Another interesting consequence of introducing central charges in the action is the possibility to derive explicitly a non trivial scalar potential V(f) which shows among other a dependence in $|Z|^2$. The component form on this scalar potential is given in Eq(4.20). Setting for example the arbitrary constant α and η_{rr} to be zero, one obtain the following expression for the scalar potential

$$V(f) = -\frac{|Z|^2}{\lambda^2}\sum_{N=0}\frac{f^{(ij)}}{\xi^{(ij)}}\times \left\{f^{0N}+\sum_{n=1}M_2(N,n)\,f^{0N-2n}\,(ff)^n\right\} \qquad (5.1)$$

Moreover its important to note that our potential share with the induced D=2 N=4 Taub-Nut hypermultiplet self interacting potential [17] namely

$$V(f) = |Z|^2 \frac{f\overline{f}}{1+\lambda f\overline{f}},$$

the dependence in $|Z|^2$, a term which we can also interpret as a BPS-like mass operator.

5. Once the two integration over the Grassmann variables and the harmonics are done, the action associated to the $(Z, \overline{Z})$ extended D=2 N=4 SU(2) Liouville self interacting model is shown to takes the form given in Eq(4.10). From this action one can explicitly derive the metric given by the tensors components explicitly presented in Eqs(4.12-18). An important remark concerning this hyper-Kahler metric is that in each term of the components $A_{ijkl}, B_{ij}, C_{ij}, D, E$ *and* F, we have a general coefficient given by

$$K(*, N, n, f) = (*)\, f^{0N} + \sum_{n=1} (*)\, f^{0N-2n} (ff)^n$$

a property which is also shown in the derived potential $V(f)$.

6. It is now widely known that harmonic superspace provides a framework for constructing general hyper-Kahler metrics. Original results in this issue are given by the Taub-Nut and Egushi-Hanson metrics exhibiting both U(1) Pauli-Gursey isometry as shown by Gibbon at all in $[11-c]$. In parallel to this work, the authors showed in an original way, the existence of the multicenter family of hyper-Kahler metrics including the above ones and their integrable deformations $[11-b, c]$. They showed also that these metrics are associated to a series of potential depending explicitly on the harmonics variables. As our D=2 N=4 SU(2) Liouville like potential Eq (2.14) share with the Egushi-Hanson potential the important feature to incorporate both the dimensionless quantity ξ^{++} which shows an explicit dependence on the harmonics, we can then conclude that our derived hyper-Kahler metric belongs to the Gibbons et al. multicenter family exhibiting a U(1) symmetry and breaking SU(2) invariance.

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