HALL EFFECT ON COUETTE FLOW WITH HEAT TRANSFER OF A DUSTY CONDUCTING FLUID IN THE PRESENCE OF UNIFORM SUCTION AND INJECTION

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Abstract

In the present study, the unsteady Couette flow with heat transfer of a dusty viscous incompressible electrically conducting fluid under the influence of a constant pressure gradient is studied without neglecting the Hall Effect. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below while the fluid is acted upon by an external uniform magnetic field applied perpendicular to the plates. The governing equations are solved numerically using finite differences to yield the velocity and temperature distributions for both the fluid and dust particles.

Keywords: *Two phase flow, dusty fluids, heat transfer, hydromagnetic flow*

1. Introduction

The importance and application of solid/fluid flows and heat transfer in petroleum transport, wastewater treatment, combustion, power plant piping, corrosive particles in engine oil flow, and many others are well known in the literature. Particularly, the flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters and has possible applications in nuclear reactors, filtration, geothermal systems, and others. The possible presence of solid particles such as ash or soot in combustion MHD generators and plasma MHD accelerators and their effect on the performance of such devices led to studies of particulate suspensions in conducting fluids in the presence of magnetic fields. For example, in an MHD generator, coal mixed with seed is fed into a combustor. The coal and seed mixture is burned in oxygen and the combustion gas expands through a nozzle before it enters the generator section. The gas mixture flowing through the MHD channel consists of a condensable vapor (slag) and a non-condensable gas mixed with seeded coal combustion products. Both the slag and the non-condensable gas are electrically conducting [1,2]. The presence of the slag and the seeded particles significantly influences the flow and heat transfer characteristics in the MHD channel. Ignoring the effect of the slag, and considering the MHD generator start-up condition, the problem reduces to unsteady two-phase flow in an MHD channel.

The hydrodynamic flow of dusty fluids was studied by a number of authors [3-7]. Later the hydromagnetic flow of dusty fluids was studied [8-12]. In the above mentioned work the Hall term was ignored in applying Ohm's law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable under these conditions, and the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term [13]. The effect of the Hall current on the Hartmann flow of a clean fluid was studied by a number of authors [14-18]. Aboul-Hassan and Attia [19] studied the influence of the Hall current on the flow and heat transfer of a dusty conducting fluid in a rectangular channel.

In the present work, the unsteady Couette flow with heat transfer of an electrically conducting, viscous, incompressible, dusty fluid is studied with the consideration of the Hall current. The fluid is assumed to be incompressible and electrically conducting and the particle phase is assumed to incompressible, electrically non-conducting dusty and pressure less. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The fluid is flowing between two infinite electrically insulating porous plates maintained at two constant but different temperatures while the particle phase is assumed to be electrically non-conducting. The fluid is subjected to a uniform suction from above and a uniform injection from below and mass conservation is assumed. An external uniform magnetic field is applied perpendicular to the plates while no electric field is applied and the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The fluid is acted upon by a constant pressure gradient. The governing equations are solved numerically using the finite difference approximations to obtain the temperature distributions for both the fluid and dust particles. The effect of the magnetic field, the Hall current and the suction velocity on both the velocity and temperature fields of the fluids as well as dust particles are reported.

2. Description of the Problem

The dusty fluid is assumed to be flowing between two infinite horizontal porous plates located at the $y=\pm h$ planes. The upper plate is moving with a constant velocity U_o while the lower plate is kept stationary. The plates are subjected to a uniform suction from above and a uniform injection from below. Thus the y-component of the velocity of the fluid is constant and denoted by v_o. The dust particles are assumed to be electrically non-conducting spherical in shape and uniformly distributed throughout the fluid and to be big enough, so that they are not pumped out through the porous plates and have no y-component of velocity. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2>T_1$. A uniform constant pressure gradient is applied in the x-direction. A uniform magnetic field B_o is applied in the positive y-direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [11]. It is required to obtain the time varying velocity and temperature distributions for both fluid and dust particles. Due to the inclusion of the Hall current term, a z-component of the velocities of the fluid and of dust particles is expected to arise. Since the plates are infinite in the x and z-directions, the physical quantities do not change in these directions that is $\partial/\partial x=\partial/\partial z=0$ and the problem is essentially one-dimensional. The governing equations for this study are based on the conservation laws of mass, linear momentum and energy of both phases.

3. The Velocity Distribution

The flow of fluid is governed by the momentum equation

$$\rho\frac{Dv}{Dt}=-\nabla P+\mu\nabla^2 v+JxB_o-KN(v-v_p) \qquad (1)$$

where ρ is the density of clean fluid, μ is the viscosity of clean fluid, v is the velocity of the fluid, $\mathbf{v}=u(y,t)\mathbf{i}+v_o\mathbf{j}+w(y,t)\mathbf{k}$, $\mathbf{v}_p$ is the velocity of dust particles, $\mathbf{v_p}=u_p(y,t)\mathbf{i}+w_p(y,t)\mathbf{k}$, **J** is the current density, N is the number of dust particles per unit volume, K is the Stokes constant $=6\pi\mu a$, and "a" is the average radius of dust particles.

The first three terms in the right-hand side of Eq. (1) are, respectively, the pressure gradient, viscosity, and Lorentz force terms. The last term represents the force due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity [3]. If the Hall term is retained, the current density **J** from the generalized Ohm's law is given by [13, 14]

$$J = \sigma\left[E + VxB_o - \beta(JxB_o)\right] \qquad (2)$$

where σ is the electric conductivity of the fluid, β is the Hall factor [13,14]. Solving Eq. (2) for **J** gives

$$JxB_o = \frac{\sigma B_o^2}{1+m^2}\left[(u+mw)i + (w-mu)k\right] \qquad (3)$$

where $m=\sigma\beta B_o$, is the Hall parameter [13,14]. Thus, in terms of Eq. (3), the two components of Eq. (1) read

$$\rho\frac{\partial u}{\partial t} + \rho v_o\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{1+m^2}(u+mw) - KN(u-u_p) \qquad (4)$$

$$\rho\frac{\partial w}{\partial t} + \rho v_o\frac{\partial w}{\partial y} = \mu\frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_o^2}{1+m^2}(w-mu) - KN(w-w_p) \qquad (5)$$

The motion of the dust particles is governed by Newton's second law applied in the x and z-directions

$$m_p\frac{\partial u_p}{\partial t} = KN(u-u_p) \qquad (6)$$

$$m_p\frac{\partial w_p}{\partial t} = KN(w-w_p) \qquad (7)$$

where m_p is the average mass of dust particles. It is assumed that the pressure gradient is applied at t=0 and the fluid starts its motion from rest. Thus,

$$t \le 0: u = u_p = w = w_p = 0 \qquad (8a)$$

For t>0, the no-slip condition at the plates implies that

$$t > 0, y = -h: u = u_p = w = w_p = 0 \qquad (8b)$$

$$t > 0, y = h: u = U_o, u_p = w = w_p = 0 \qquad (8c)$$

4. The Temperature Distribution

Heat transfer takes place from the upper hot plate to the lower cold plate by conduction through the fluid. Since the hot plate is above, there is no natural convection; however, there is a forced convection due to the suction and injection. In addition to the heat transfer, there is a heat generation due to both the Joule and viscous dissipations. The dust particles gain heat from the fluid by conduction through their spherical surface. Since, the problem deals with a two-phase flow, two energy equations are required [13,21]. The energy equations describing the temperature distributions for both the fluid and dust particles read

$$\rho c\frac{\partial T}{\partial t} + \rho c v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma B_o^2}{1+m^2}(u^2+w^2) + \frac{\rho_p c_s}{\gamma_T}(T_p - T), \qquad (9)$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma_T}(T_p - T), \qquad (10)$$

where T is the temperature of the fluid, T_p is the temperature of the particles, c is the specific heat capacity of the fluid at constant volume, k is the thermal conductivity of the fluid, ρ_p is the mass of dust particles per unit volume of the fluid, γ_T is the temperature relaxation time, and c_s is the specific heat capacity of the particles.

The last three terms on the right-hand side of Eq. (9) represent the viscous dissipation, the Joule dissipation (j^2/σ), and the heat conduction between the fluid and dust particles respectively. The temperature relaxation time depends, in general, on the geometry, and since the dust particles are assumed to be spherical in shape, the last term in Eq. (9) is equal to $4\pi aNk(T_p-T)$. Hence

$$\gamma_T = \frac{3\Pr \gamma_p c_s}{2c}$$

where γ_p is the velocity relaxation time $=2\rho_s a^2/9\mu$, Pr is the Prandtl number $=\mu c/k$, and ρ_s is the material density of dust particles $=3\rho_p/4\pi a^3 N$.

T and T_p must satisfy the initial and boundary conditions

$$t \le 0 : T = T_p = T_1, \qquad (11a)$$

$$t > 0, y = -h : T = T_p = T_1, \qquad (11b)$$

$$t > 0, y = h : T = T_p = T_2. \qquad (11c)$$

Equations (4)-(11) can be made dimensionless by introducing the following dimensionless variables and parameters

$$(\hat{x},\hat{y}) = \frac{(x,y)}{h}, \hat{t} = \frac{tU_o}{h}, (\hat{u},\hat{w}) = \frac{(u,w)}{U_o}, (\hat{u}_p,\hat{w}_p) = \frac{(u_p,w_p)}{U_o}, \hat{P} = \frac{P}{\rho U_o^2},$$

$$\hat{T} = \frac{T - T_1}{T_2 - T_1}, \hat{T}_p = \frac{T_p - T_1}{T_2 - T_1}$$

$S = v_o / U_o$, the suction parameter, $\mathrm{Re} = U_o \rho h / \mu$, is the Reynolds number, $Ha = B_o h\sqrt{\sigma/\mu}$, the Hartmann number, $E_c = U_o^2 / c(T_2 - T_1)$, the Eckert number, $G = m_p \mu / \rho h^2 K$, is the particle mass parameter, $R = KNh^2/\mu$ is the particle concentration parameter. $L_o = \rho h^2 / \mu\gamma_T$ is the temperature relaxation time parameter.

In terms of the above dimensionless quantities Eqs. (4)-(11) read

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{1}{\mathrm{Re}}\frac{dP}{dx} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} - \frac{Ha^2}{\mathrm{Re}(1+m^2)}(u + mw) - \frac{R}{\mathrm{Re}}(u - u_p) \qquad (12)$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = \frac{1}{\mathrm{Re}}\frac{\partial^2 w}{\partial y^2} - \frac{Ha^2}{\mathrm{Re}(1+m^2)}(w - mu) - \frac{R}{\mathrm{Re}}(w - w_p) \qquad (13)$$

$$G\frac{\partial u_p}{\partial t} = u - u_p \qquad (14)$$

$$G\frac{\partial w_p}{\partial t} = w - w_p \qquad (15)$$

$$t \le 0 : u = u_p = w = w_p = 0 \qquad (16a)$$

$$t > 0, y = -h : u = u_p = w = w_p = 0 \qquad (16b)$$

$$t > 0, y = h : u = 1, u_p = w = w_p = 0 \qquad (16c)$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Re}\Pr}\frac{\partial^2 T}{\partial y^2} + \frac{E_c}{\mathrm{Re}}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{Ha^2 E_c}{\mathrm{Re}(1+m^2)}(u^2 + w^2)$$
$$+ \frac{2R}{3\mathrm{Re}\Pr}(T_p - T), \qquad (17)$$

$$\frac{\partial T_p}{\partial t} = -L_o(T_p - T), \qquad (18)$$

$$t \le 0 : T = T_p = 0, \tag{19a}$$

$$t > 0, y = -1 : T = T_p = 0, \tag{19b}$$

$$t > 0, y = 1 : T = T_p = 1. \tag{19c}$$

where the hats are dropped for convenience.

Equations (12)-(19) represent a system of partial differential equations which is solved numerically using the finite difference approximation. The Crank-Nicolson implicit method [22] is used at two successive time levels. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximation in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm [22]. Computations have been made for dP/dx=5, Re=1, R=0.5, G=0.8, L_o=0.7, Pr=1, and E_c=0.2.

5. Results and Discussion

Figures 1-3 present, respectively, the profiles of the velocity components u, u_p, w and u_p w_p and temperatures T and T_p for various values of time t. The figures are plotted for Ha=1, m=3 and S=1. It is observed from Figs. 1a, 2a, and 3a that the velocity component u reaches the steady state faster than w which, in turn, reaches the steady state faster then T. This is expected, since u is the source of w, while both u and w act as sources for the temperature. The same observation is clear in Figs. 1b, 2b, and 3b for u_p, w_p and T_p, respectively. Comparing Figs. 1a, 2a and 3a with 1b, 2b and 3b, respectively, shows that the velocity components and temperature of the fluid phase reach the steady state faster than that of the particle phase. This is because the fluid velocity is the source for the dust particles' velocity.

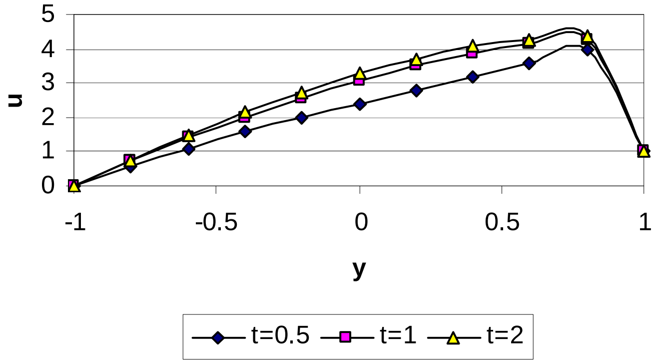
Figures 4-6 show the time evolution of the velocity components and temperature at the centre of the channel (y=0), respectively, for the fluid and particle phases for various values of the Hall parameter m and for Ha=1 and S=0. It is clear from Figs. 4a and 4b that increasing the parameter m increases u and u_p. This is because the effective conductivity ($\sigma/(1+m^2)$) decreases with increasing m which reduces the magnetic damping force on u and consequently u and u_p increase. In Figs. 5a and 5b, the velocity components w and w_p increases with increasing m for small values of m (m=0 to 1). For large values of m (m>1), the velocities decrease with m. To explain these observations, we argue that the velocity component u is the source of w, which is in turn the source for u_p. The source term is proportional to $mu/(1+m^2)$, where u depends implicitly on m. It is clear that the source term increases or decreases with m according to whether m is small or large. Figures 6a and 6b indicate that increasing m decreases T and T_p for all t. This can be attributed to the fact that an increase in m decreases the Joule dissipation which is proportional to ($1/(1+m^2)$). Comparing Figs. 6a and 6b ensures that the temperature of the fluid reaches the steady state faster than the temperature of the particles.

Figures 7-9 show the time evolution of the velocity components and temperature at the centre of the channel (y=0), respectively, for the fluid and particle phases for various values of the Hartmann number Ha and for m=3 and S=0. Figures 7a and 7b indicate that increasing Ha decreases u and u_p as a result of increasing damping force on u. Figures 8a and 8b ensure that increasing Ha increases w and w_p for all values of Ha due to the effect of Ha in decreasing u which decreases the damping force on w. Figures 9a and 9b show that the increasing Ha increases T and T_p as a result of increasing the Joule dissipations.

Figures 10-12 present the time evolution of the velocity components and temperature at the centre of the channel (y=0), respectively, for the fluid and particle phases for various values of the suction parameter S and for Ha=1 and m=3. Figures 10a, 10b, 11a, and 11b show that increasing the suction decreases u, w, u_p and w_p and their steady state times due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. Figures 12a and 12b show that increasing S decreases the temperatures T and T_p at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel.

6. Conclusion

The unsteady Couette flow with heat transfer of a dusty conducting fluid under the influence of an applied uniform magnetic field has been studied, considering the Hall effect in the presence of uniform suction and injection. The effect of the magnetic field, the Hall parameter, and the suction and injection velocity on the velocity and temperature distributions for both the fluid and particle phases has been investigated. It was found that both the fluid and the solid-particle phases have two components of velocity. The main two components of velocity of the fluid and dust particles u and u_p, respectively, were found to increase with increase in the Hall parameter m. However, the other two components of velocity w and w_p which result due to the Hall effect increase with the Hall parameter m for small m and decrease with m for large values of m. It was found also that the temperatures of both fluid and particle phases deceases with the Hall parameter m.



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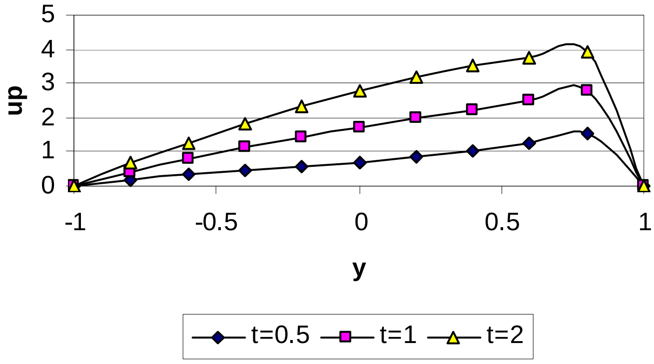
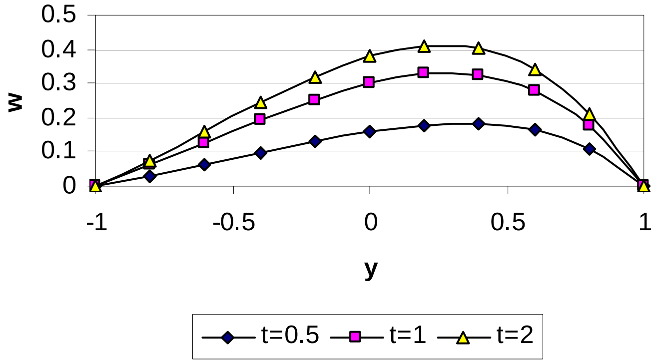


(b)

Fig. 1 Time variation of the profile of: (a) u and (b) u_p. (Ha=1, m=3, and S=1)



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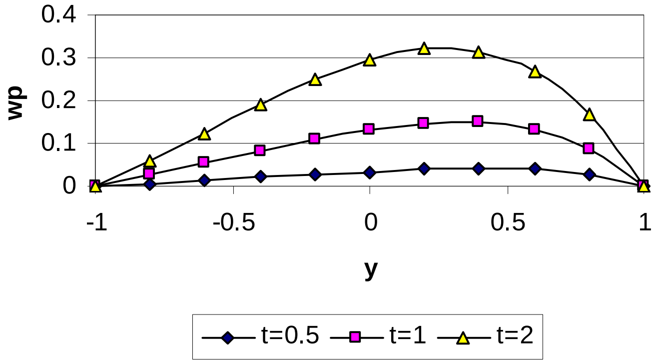

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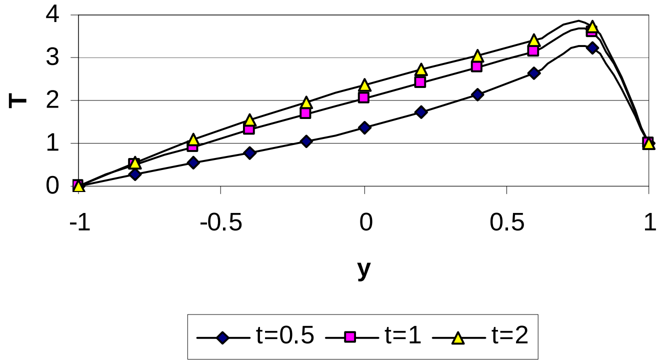
Fig. 2 Time variation of the profile of: (a) w and (b) w_p. (Ha=1, m=3, and S=1)



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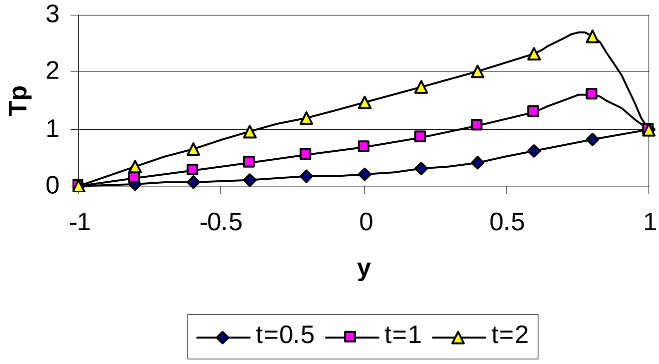

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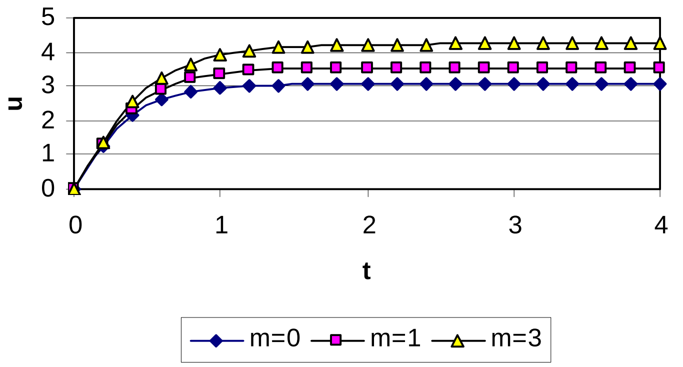
Fig. 3 Time variation of the profile of: (a) T and (b) T_p. (Ha=1, m=3, and S=1)



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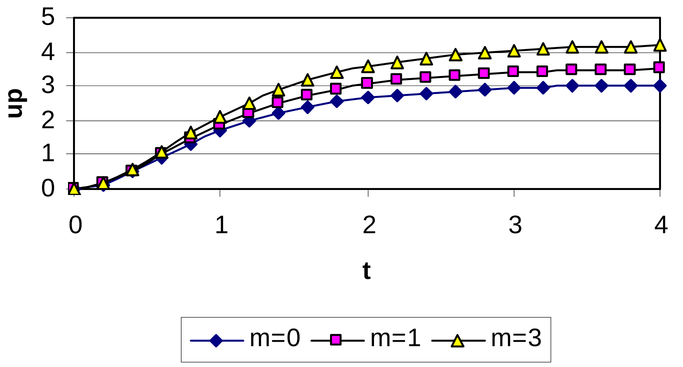

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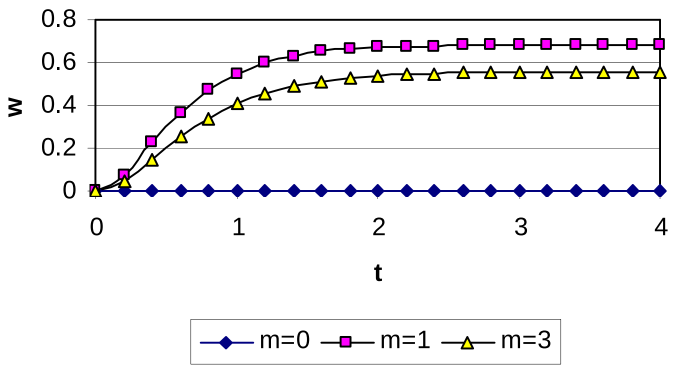
Fig. 4 Effect of the parameter m on the time variation of : (a) u at y=0 and (b) u_p at y=0. (Ha=1 and S=0)



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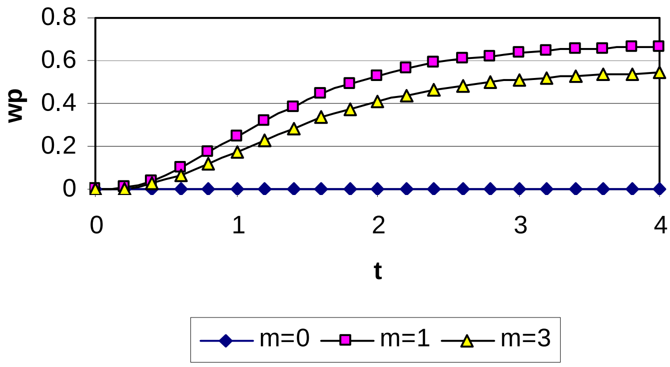

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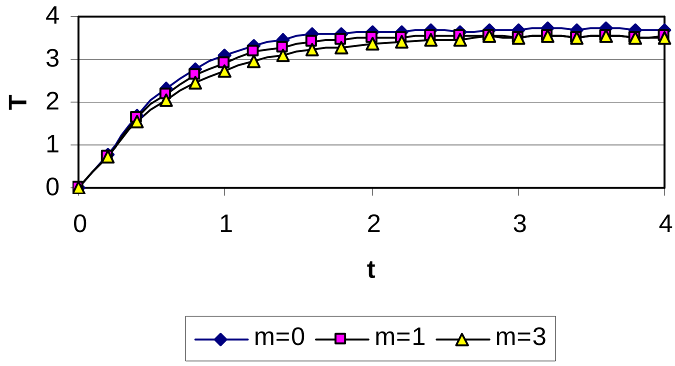
Fig. 5 Effect of the parameter m on the time variation of : (a) w at y=0 and (b) w_p at y=0. (Ha=1 and S=0)



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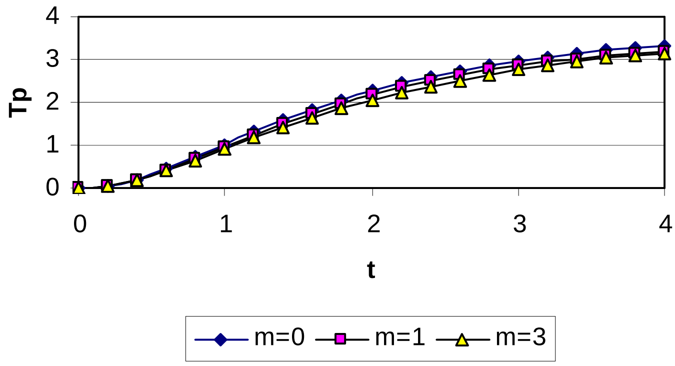

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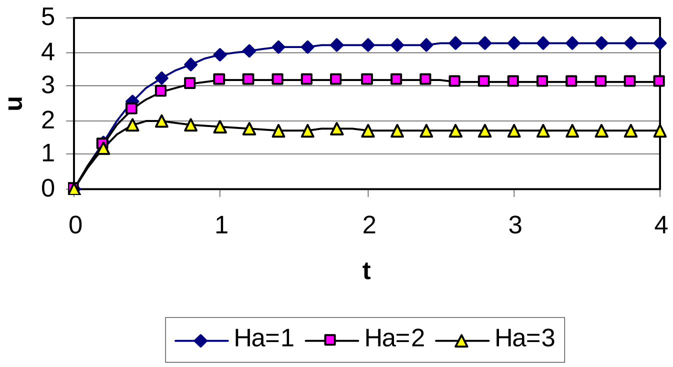
Fig. 6 Effect of the parameter m on the time variation of : (a) T at y=0 and (b) T_p at y=0. (Ha=1 and S=0)



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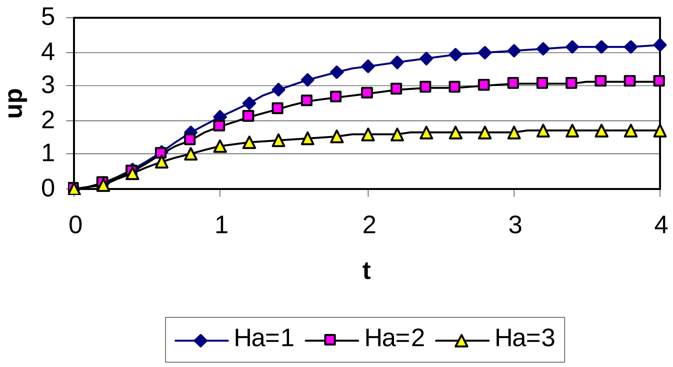

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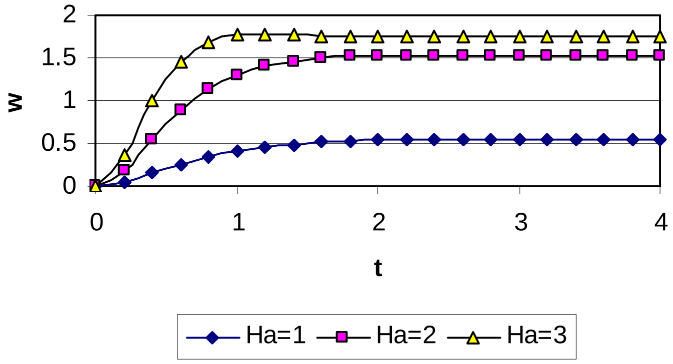
Fig. 7 Effect of the parameter m on the time variation of : (a) u at y=0 and (b) u_p at y=0. (m=3 and S=0)



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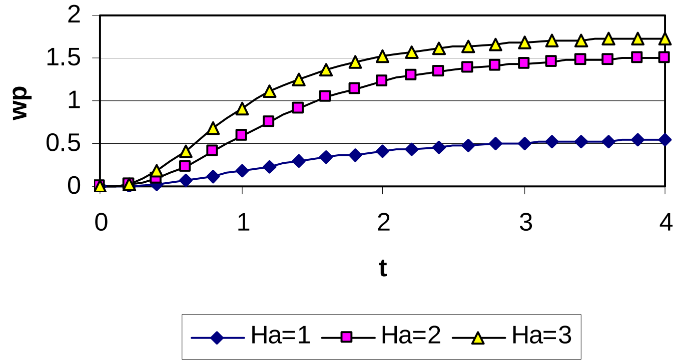

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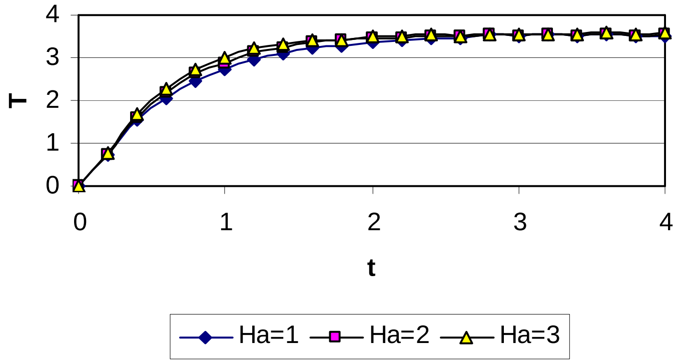
Fig. 8 Effect of the parameter m on the time variation of : (a) w at y=0 and (b) w_p at y=0. (m=3 and S=0)



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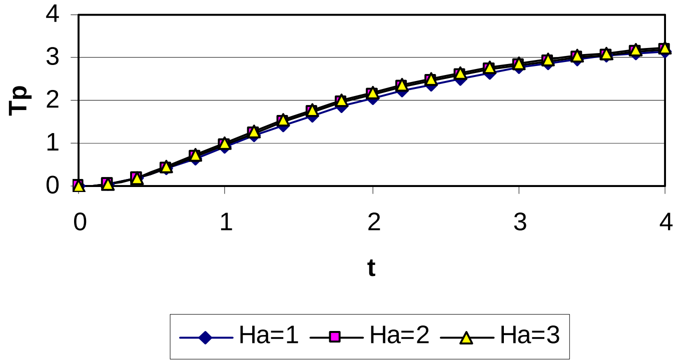

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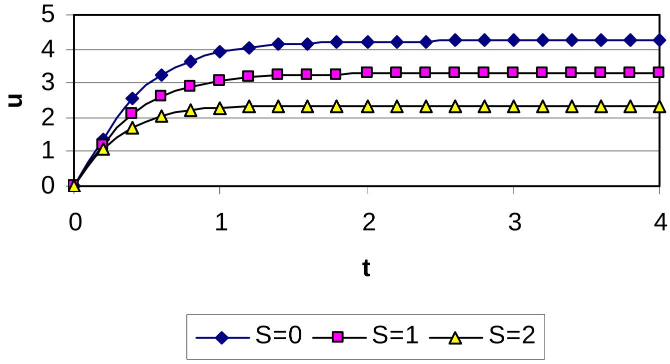
Fig. 9 Effect of the parameter m on the time variation of : (a) T at y=0 and (b) T_p at y=0. (m=3 and S=0)



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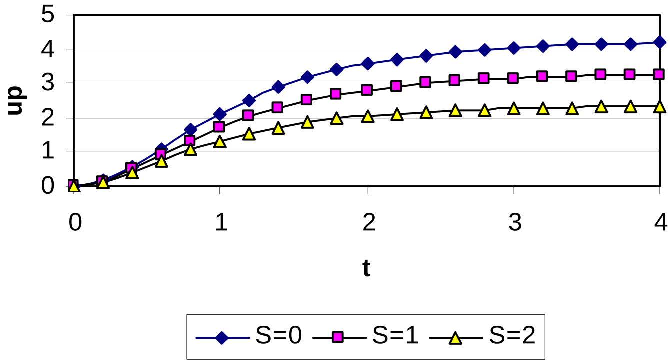

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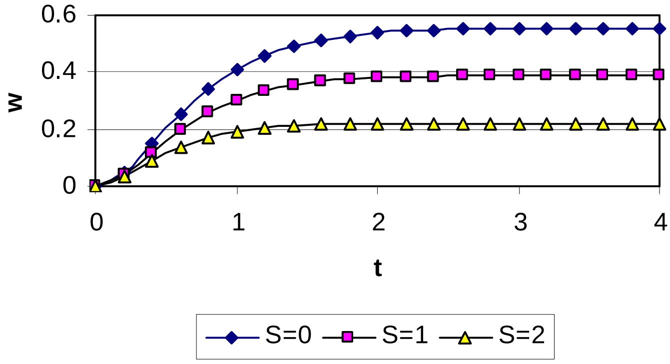
Fig. 10 Effect of the parameter m on the time variation of : (a) u at y=0 and (b) u_p at y=0. (m=3 and Ha=1)



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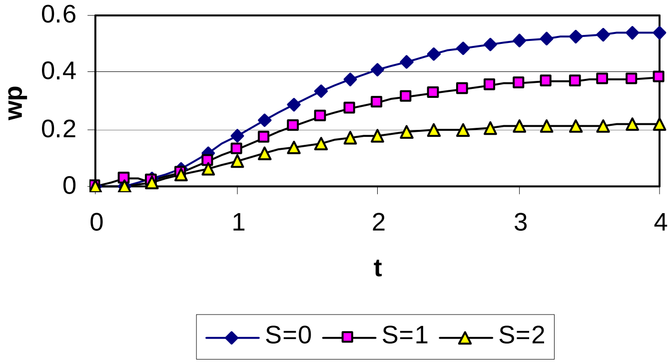

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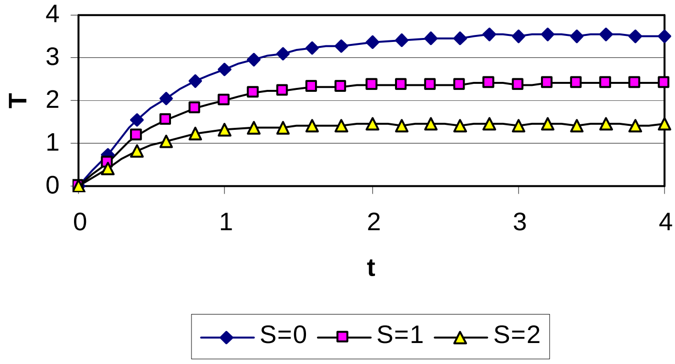
Fig. 11 Effect of the parameter m on the time variation of : (a) w at y=0 and (b) w_p at y=0. (m=3 and Ha=1)



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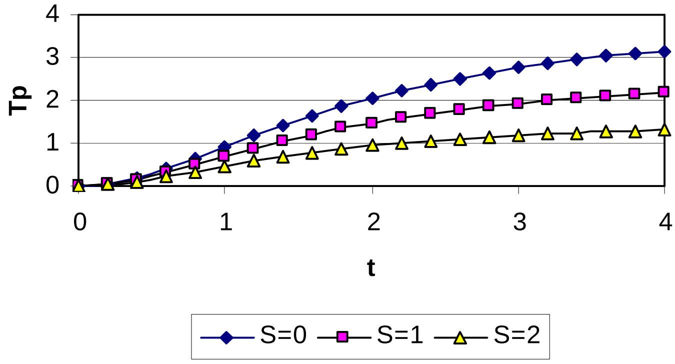

(b)
Fig. 12 Effect of the parameter m on the time variation of : (a) T at y=0 and (b) T_p at y=0. (m=3 and Ha=1)

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