African Journal Of Mathematical Physics Vol **3** *No 1 (2006)55-63*

Approach Unifying Spins and Charges in d=14 Predicts Properties of Four Families of Quarks and Leptons

N. Mankoč Borštnik

Department of physics, University of Ljubljana, Jadranska 19, 1111

Abstract

Ten years ago the approach unifying all the internal degrees of freedom - that is the spin and all the charges into only the spin - was proposed[1−10]. In this approach spinors, living in d $(= 1 + 13)-$dimensional space, carry only the spin and interact with only the gravity through spin connections and vielbeins. After assuming appropriate breaks of symmetries a spin manifests in $d = (1 + 3)$ "physical" space as the spin and all the known charges, while the covariant derivative of a spinor in $d(= 1 + 13)-$ manifests as the observable covariant derivative in $d = 1 + 3$ and as the mass terms, leading to masses of four families of quarks and leptons and to the corresponding mixing matrices.

INTRODUCTION

The Standard model of the electroweak and strong interactions (extended by the inclusion of the massive neutrinos) fits well all the existing experimental data. It assumes around 25 parameters and requests, the origins of which is not yet understood.

The advantage of the approach unifying spins and charges[1−10] is, that it might offer possible answers to the open questions of the Standard electroweak model. We demonstrated in references[6,8−10] that a left handed $SO(1, 13)$ Weyl spinor multiplet includes, if the representation is interpreted in terms of the subgroups $SO(1, 3)$, $SU(2)$, $SU(3)$ and the sum of the two $U(1)$'s, all the spinors of the Standard model - that is the left handed $SU(2)$ doublets and the right handed $SU(2)$ singlets of (with the group $SU(3)$ charged) quarks and (chargeless) leptons. Right handed neutrinos - weak and hyper chargeless - are also included. In the gauge theory of gravity (in our case in $d = (1 + 13)$-dimensional space), the Poincaré group is gauged, leading to spin connections and vielbeins, which determine the gravitational field[2,8]. There are the spin connection and vielbein fields, which manifest - after the appropriate compactification (or some other kind of making the rest of d-4 space unobservable at low energies) - in the four dimensional space as all the gauge fields of the known charges, as well as determine the mass matrices of quarks and leptons, that is the Yukawa couplings of the Standard model.

The approach seems to have, like all the Kaluza-Klein-like theories, a very serious disadvantage, namely that there might not exist any massless, mass protected spinors, which are, after the break of symmetries, chirally coupled to the desired (Kaluza-Klein) gauge fields[11]. This would mean that there are no observable spinors at low energies. In the reference[12] we present a toy model, which leads to massless, mass protected spinors, chirally coupled to the Kaluza-Klein gauge fields, observable at low energies. Although not yet realistic, the toy model looks promising.

In refs.[9,10,17,18] it was shown, that the approach unifying spins and charges might explain the Yukawa couplings within one family, as well as among families, if extended on two types of the Clifford algebra objects. For the appearance of families, namely, the second kind of the Clifford algebra objects iA responsible. an even number of families was predicted, in particular, four families of quarks and leptons. In this talk the approach of unifying spins and charges is briefly presented nd the mass matrices are derived from the starting Lagrange density and presented in terms of the spin connection fields of two types, after making several assumptions to simplify the results and make them calculable.

I. WEYL SPINORS IN $D = (1 + 13)$ MANIFESTING FAMILIES OF QUARKS AND LEPTONS IN $D = (1 + 3)$

We start with a left handed Weyl spinor in (1 + 13)-dimensional space. A spinor carries only the spin (no charges) and interacts accordingly with only the gauge gravitational fields - with their spin connections and vielbeins. We allow families of left handed spinors, since we assume indeed two kinds of the gauge fields[1−10]. One kind is the ordinary gauge field (gauging the Poincaré symmetry in $d = 1+13$). The corresponding spin connection field appears for spinors as a gauge field of $S^{ab} = \frac{1}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$, where γ^a are the ordinary Dirac operators in d-dimensional space. It contributes to all known gauge fields and to the diagonal elements of mass matrices (connecting right handed weak chargeless quarks or leptons with left handed weak charged partners within one family of spinors).

The second kind of gauge fields is in our approach responsible for the Yukawa couplings among families of spinors and might explain the origin of the families of quarks and leptons. The corresponding spin connection field appears for spinors as a gauge field of $\tilde{S}^{ab}$ ($\tilde{S}^{ab} = \frac{1}{2}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$) with $\tilde{\gamma}^a$, which are the Clifford algebra objects[14], like γ^a, but anti commute with γ^a.

Accordingly we write the action for a Weyl (massless) spinor in $d(= 1 + 13)$ - dimensional space as follows[1]

$$S = \int d^dx \; \frac{1}{2}(e\bar{\psi}\gamma^a p_{0a}\psi) + h.c., \tag{1.1}$$

with $p_{0a} = f^\alpha{}_a p_{0\alpha}$ and $p_{0\alpha} = p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}$. Here $f^\alpha{}_a$ are vielbeins (inverted to the gauge field of the generators of translations $e^a{}_\alpha$, $e^a{}_\alpha f^\alpha{}_b = \delta^a_b$, $e^a{}_\alpha f^\beta{}_a = \delta_\alpha{}^\beta$), with $e = det(e^a{}_\alpha)$, while $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ are the two kinds of the spin connection fields, the gauge fields of S^{ab} and $\tilde{S}^{ab}$, respectively, corresponding to the two kinds of the Clifford algebra objects[13,10], namely γ^a and $\tilde{\gamma}^a$ ($\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$, $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$). To see that one Weyl spinor in $d = (1 + 13)$ with the spin as the only internal degree of freedom, can manifest in our-dimensional "physical" space as the ordinary ($SO(1, 3)$) spinor with all the known charges of one family of quarks and leptons of the Standard model, one has to analyze one Weyl spinor (we make a choice of the left handed one) representation in terms of the subgroups $SO(1, 3) \times U(1) \times SU(2) \times SU(3)$ (the reader can see this analyses in several references, like the one[10].)

To see that the Yukawa couplings are the part of the starting Lagrangian of eq.(1.1), we rewrite the Lagrangian in eq.(1.1) as follows[10]

---

[1]latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an einstein index (a curved index). letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). we assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

$$\mathcal{L} = \bar{\psi}\gamma^\mu (p_\mu - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_\mu)\psi + \\ 2i\psi^+ S^{0s} S^{tt'} \omega_{tt's}\psi + 2i\psi^+ S^{0s} \tilde{S}^{tt'} \tilde{\omega}_{tt's}\psi + \text{the rest.} \tag{1.2}$$

index A determines the charge groups ($SU(3), SU(2)$ and the two $U(1)$'s), index i determines the generators within one charge group. τ^{Ai} denote the generators of the charge groups (expressible[8] in terms of S^{st}, $s,t \in 5,6,..,14$), while $A_\mu, \mu = 0,1,2,3$, denote the corresponding gauge fields (expressible in terms of $\omega_{st\mu}$).

The second and the third term look like mass terms, since $\omega_{tt's}$ $(= f^\sigma_s \omega_{tt'\sigma})$ and $\tilde{\omega}_{tt's}$, $s,t \in 5,6,7,8$, $\sigma \in (5),(6),(7),(8))$, behave in $d(=1+3)-$ dimensional space like scalar fields, while the operator $S^{0s}, s = 7,8$, for example, transforms a right handed weak chargeless spinor (for example u_R) into a left handed weak charged spinor (in this case to u_L), without changing the spin in $d = 1+3$ - just what the Yukawa couplings with the Higgs doublet included do in the Standard model formulation. It should be pointed out that no Higgs weak charge doublet is needed here, as $S^{0s}, s = 7,8$ does his job.

There are several ways of breaking the group $SO(1,13)$ down to subgroups of the Standard model. (One of) the most probable breaks, suggested by the approach unifying spins and charges, is the following one

$$\begin{array}{c} \mathrm{SO}(1,13) \\ \downarrow \\ \underbrace{\mathrm{SO}(1,7) \otimes \mathrm{SU}(3) \otimes \mathrm{U}(1)} \\ \\ \downarrow \\ \mathrm{SO}(1,3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1) \otimes \mathrm{SU}(3) \end{array}$$

We start from a massless left handed Weyl spinor in $d = 1+13$. we assume that the first break of symmetry leads again to massless spinors in $d = 1+7$, chirally coupled with the $SU(3)$ and $U(1)$ charge to the corresponding fields, which follow from the spin connection and vielbein fields in $d = 1+13$.

A. Spin and charges of one left handed fundamental representation in SO(1,13)

The group $SO(1,13)$ of the rank 7 has as possible subgroups the groups $SO(1,3)$, $SU(2), SU(3)$ and the two $U(1)$'s, with the sum of the ranks of all these subgroups equal to 7. These subgroups are candidates for describing the spin, the weak charge, the colour charge and the two hyper charges, respectively (only one is needed in the Standard model). The generators of these groups can be written in terms of the generators S^{ab} as follows

$$\tau^{Ai} = \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}, \quad \{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}. \tag{1.3}$$

The weak charge ($SU(2)$ with the generators τ^{1i}) and one $U(1)$ charge (with the generator τ^{21}) content of the compact group $SO(4)$ (a subgroup of $SO(1,13)$) can be demonstrated when expressing

$$\tau^{11} := \frac{1}{2}(\mathcal{S}^{58} - \mathcal{S}^{67}), \quad \tau^{12} := \frac{1}{2}(\mathcal{S}^{57} + \mathcal{S}^{68}), \quad \tau^{13} := \frac{1}{2}(\mathcal{S}^{56} - \mathcal{S}^{78}) \\ \tau^{21} := \frac{1}{2}(\mathcal{S}^{56} + \mathcal{S}^{78}). \tag{1.4}$$

To see the colour charge and one additional $U(1)$ content in the group $SO,(1,13)$ we write τ^{3i} and τ^{41}, respectively in terms of the generators $\mathcal{S}^{ab}$

$$\tau^{31} : = \frac{1}{2}(\mathcal{S}^{9\,12} - \mathcal{S}^{10\,11}), \quad \tau^{32} := \frac{1}{2}(\mathcal{S}^{9\,11} + \mathcal{S}^{10\,12}), \quad \tau^{33} := \frac{1}{2}(\mathcal{S}^{9\,10} - \mathcal{S}^{11\,12}),$$

$$\begin{aligned}
\tau^{34} : &= \frac{1}{2}(\mathcal{S}^{9\,14} - \mathcal{S}^{10\,13}), \quad \tau^{35} := \frac{1}{2}(\mathcal{S}^{9\,13} + \mathcal{S}^{10\,14}), \quad \tau^{36} := \frac{1}{2}(\mathcal{S}^{11\,14} - \mathcal{S}^{12\,13}),\\
\tau^{37} : &= \frac{1}{2}(\mathcal{S}^{11\,13} + \mathcal{S}^{12\,14}), \quad \tau^{38} := \frac{1}{2\sqrt{3}}(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} - 2\mathcal{S}^{13\,14})\\
\tau^{41} : &= -\frac{1}{3}(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} + \mathcal{S}^{13\,14}).
\end{aligned} \tag{1.5}$$

To reproduce the Standard model groups one must introduce the two superpositions of the two $Y(1)$'s generators as follows

$$Y = \tau^{41} + \tau^{21}, \quad Y' = \tau^{41} - \tau^{21}. \tag{1.6}$$

B. Spinor representation in terms of Clifford algebra objects

We start by defining two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$, with the properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0. \tag{1.7}$$

The operators $\tilde{\gamma}^a$ are introduced formally as operating on any Clifford algebra object B from the left hand side, but they also can be expressed in terms of the ordinary γ^a as operating from the right hand side as follows $\tilde{\gamma}^a B := i(-)^{n_b} B\gamma^b$, with $(-)^{n_b} = +1$ or -1, when the object B has a Clifford even or odd character, respectively. accordingly two kinds of generators of the Lorentz transformations follow, namely $S^{ab} = i/4(\gamma^a\gamma^b - \gamma^b\gamma^a)$ and $\tilde{S}^{ab} = i/4(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, with the property $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$.

We define spinor representations as eigen states of the chosen Cartan sub algebra of the Lorentz algebra $SO(1,13)$, with the operators S^{ab} and $\tilde{S}^{ab}$ in the two Cartan sub algebra sets, with the same indices in both cases. By introducing the notation

$$\begin{aligned}
\stackrel{ab}{(\pm i)}: &= \frac{1}{2}(\gamma^a \mp \gamma^b), \quad \stackrel{ab}{[\pm i]}:= \frac{1}{2}(1 \pm \gamma^a\gamma^b), \; for\; \eta^{aa}\eta^{bb} = -1,\\
\stackrel{ab}{(\pm)}: &= \frac{1}{2}(\gamma^a \pm i\gamma^b), \quad \stackrel{ab}{[\pm]}:= \frac{1}{2}(1 \pm i\gamma^a\gamma^b), \; for\; \eta^{aa}\eta^{bb} = 1,
\end{aligned} \tag{1.8}$$

it can be shown that

$$\begin{aligned}
S^{ab} \stackrel{ab}{(k)}: &= \frac{k}{2} \stackrel{ab}{(k)}, \quad S^{ab} \stackrel{ab}{[k]}:= \frac{k}{2} \stackrel{ab}{[k]},\\
\tilde{S}^{ab} \stackrel{ab}{(k)}: &= \frac{k}{2} \stackrel{ab}{(k)}, \quad \tilde{S}^{ab} \stackrel{ab}{[k]}:= -\frac{k}{2} \stackrel{ab}{[k]}.
\end{aligned} \tag{1.9}$$

The above binomials are all "eigenvectors" of the generators S^{ab}, as well as of $\tilde{S}^{ab}$. We shall make use of the relations

$$\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0, \quad \stackrel{ab}{(k)}\stackrel{ab}{(-k)}= \eta^{aa} \stackrel{ab}{[k]}, \quad \stackrel{ab}{[k]}\stackrel{ab}{[k]}=\stackrel{ab}{[k]}, \quad \stackrel{ab}{[k]}\stackrel{ab}{[-k]}= 0,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0, \quad \stackrel{ab}{[k]}\stackrel{ab}{(k)}=\stackrel{ab}{(k)}, \quad \stackrel{ab}{(k)}\stackrel{ab}{[-k]}=\stackrel{ab}{(k)}, \quad \stackrel{ab}{[k]}\stackrel{ab}{(-k)}= 0.
\end{aligned} \tag{1.10}$$

The reader should notice that γ^a's transform the binomial $\stackrel{ab}{(k)}$ into the binomial $\stackrel{ab}{[-k]}$, whose eigen value with respect to S^{ab} change sign, while $\tilde{\gamma}^a$'s transform the binomial $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$ with unchanged "eigen value" with respect to S^{ab}.

We define the operators of handedness of the group $SO(1,13)$ and of the subgroups $SO(1,3), SO(1,7)$ and $SO(4)$ as follows: $\Gamma^{(1,13)} = i2^7\; S^{03}S^{12}S^{56}...S^{13\,14}$, $\Gamma^{(1,3)} = -i2^2 S^{03}S^{12}, \Gamma^{(1,7)} = -i2^4 S^{03}S^{12}S^{56}S^{78}, \Gamma^{(4)} = 2^2 S^{56}S^{78}$. In order to represent one Weyl left handed spinor as products of binomials $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, one must make a choice of the operators belonging to the Cartan sub algebra of 7 elements of $SO(1,13)$. We make the following choice $S^{03}, S^{12}, S^{56}, \cdots, S^{13\,14}$. We make a choice of a starting state of one Weyl representation of the group $SO(1,13)$, which is the eigen state of all the members of the Cartan sub algebra and is left handed ($\Gamma^{(1,13)} = -1$)

$$\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} |\psi\rangle =$$
$$(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)|(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8)||(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle. \tag{1.11}$$

One easily finds that the eigen values of the chosen Cartan sub algebra elements are $+i/2, 1/2, 1/2, 1/2, 1/2, -1/2, -1/2$, respectively. This state is a right handed spinor with respect to $SO(1,3)$ ($\Gamma^{(1,3)} = 1$), with spin up ($S^{12} = 1/2$), it is $SU(2)$ singlet ($\tau^{13} = 0$, eq.(1.3)), and it is the member of the $SU(3)$ triplet with ($\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$), it has $\tau^{21} = 1/2$ and $\tau^{41} = 1/6$, while $Y = 2/3$ and $Y' = -1/3$. The starting state (eq.(1.11)) can be recognized in terms of the Standard model subgroups as the right handed weak chargeless u-quark carrying one of the three colours.

To obtain all the states of one Weyl spinor, one only has to apply on the starting state of eq.(1.11) the generators S^{ab}. All the quarks and the leptons of one family of the Standard model appear in this multiplet. We present in Table I all the quarks of one particular colour (the right handed weak chargeless u_R, d_R and left handed weak charged u_L, d_L, with the colour $(1/2, 1/(2\sqrt{3}))$ in the Standard model notation). They all are members of one $SO(1,7)$ multiplet (and belong to one Weyl left handed representation in $d = 1 + 13$).

i		$\vert^a\psi_i>$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{21}	τ^{33}	τ^{38}	τ^{41}	Y	Y'
		octet, $\Gamma^{(1,7)} = 1$, $\Gamma^{(6)} = -1$, of quarks										
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	1/2	1	0	1/2	1/2	$1/(2\sqrt{3})$	1/6	2/3	-1/3
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	-1/2	1	0	1/2	1/2	$1/(2\sqrt{3})$	1/6	2/3	-1/3
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	1/2	1	0	-1/2	1/2	$1/(2\sqrt{3})$	1/6	-1/3	2/3
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	-1/2	1	0	-1/2	1/2	$1/(2\sqrt{3})$	1/6	-1/3	2/3
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	1/2	-1	-1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	-1/2	-1	-1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	1/2	-1	1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	-1/2	-1	1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6

Table I. The 8-plet of quarks - the members of $SO(1,7)$ subgroup, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1,13)$. It contains left handed weak charged quarks and right handed weak chargeless quarks of a particular colour ($(1/2, 1/(2\sqrt{3}))$). The reader can find the whole Weyl representation in the ref.[10].

C. Appearance of families

While the generators of the Lorentz group S^{ab}, with a pair of (ab), which does not belong to the Cartan sub algebra, transform one vector of one Weyl representation into another, transform the generators $\tilde{S}^{ab}$ (again if the pair (ab) does not belong to the Cartan set) a member of one family into the same member of another family, leaving all the other quantum numbers (determined by S^{ab}) unchanged[1,2,4,8,7,10]. The operator $\tilde{\gamma}^a$ changes $\stackrel{ab}{(+)}$ (or $\stackrel{ab}{(+i)}$) into $\stackrel{ab}{[+]}$ (or into $\stackrel{ab}{[+i]}$, respectively), without changing the "eigen values" of the Cartan sub algebra set of the operators S^{ab} as one can see bellow

$$\overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \mid\mid \overset{910}{(+i)}\overset{1112}{(-)}\overset{1314}{(-)}$$
$$\overset{03}{[+i]}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{(+)} \mid\mid \overset{910}{(+i)}\overset{1112}{(-)}\overset{1314}{(-)} \,. \tag{1.12}$$

One can easily see that both states of (1.12) describe a right handed u-quark of the same colour. They are equivalent with respect to the operators S^{ab}.

II. MASS MATRICES IN THE APPROACH UNIFYING SPINS AND CHARGES

Let us look at the Lagrange density in $d = 1+7$ for one Weyl spinor of the left handedness and of all possible $SU(3)\times U(1)$ charges. The Lagrange density manifests the coupling of a spinor to the colour, the weak and the hyper charge gauge fields, as well as the masses. We recognize, that the terms with $\gamma^0\gamma^7$, for example, transform a right handed weak chargeless spinors with the spin $1/2$ (like the quark u_R from the first row in Table I) into a left handed weak charged spinors with the spin $1/2$ (in our example into the quark u_L from the seventh's row in Table I)

$$\mathcal{L}_{\Updownarrow} = \bar{\psi}\gamma^m (p_m - \sum_{a,i} \tau^{Ai} A^{Ai}_m)\ \psi +$$
$$+\psi^{+}\gamma^0\gamma^s\ (p_s - \sum_{a,i} \tau^{Ai} A^{Ai}{}_s)\psi + \text{terms with } \tilde{S}^{ab}. \tag{2.1}$$

We can rearrange the first term in the Lagrangian of eq.(2.1), with $\mu =\in \{0,1,2,3\}$, to manifest the Standard model structure $\mathcal{L}_f = \bar{\psi}\gamma^\mu \{p_\mu - \frac{g}{2}(\tau^+ W^+{}_\mu + \tau^- W^-{}_\mu) + \frac{g^2}{\sqrt{g^2+g'^2}} Q' Z_\mu + + \frac{gg'}{\sqrt{g^2+g'^2}} Q A_\mu + \sum_i g^c \tau^{3i} A^{3i}{}_\mu + A^{Y'}{}_\mu\}\ \psi$, with $Q = \tau^{13} + Y = S^{56} + \tau^{41}, Q' = \tau^{13} - (\frac{g'}{g})^2 Y = \frac{1}{2}(1-(\frac{g'}{g})^2)S^{56} - \frac{1}{2}(1+(\frac{g'}{g})^2)S^{78} - (\frac{g'}{g})^2\tau^{41}$. In order to be in agreement with the observations, we assume that the term $A^{Y'}{}_\mu$ is negligible at low energies. If we define the following superposition of the operators $(\gamma^7 \pm i\gamma^8) = 2\ \overset{78}{(\pm)}$ and the fields $A^y_{\pm} = -\frac{1}{4}(A^y{}_7 \mp A^y{}_8)$, with $y = Y, Y'$, we end up with a simple expression for the contribution of the operators S^{ab} to the mass matrices

$$\mathcal{L} = \psi^{+}\gamma^0 \sum_{y=Y,Y'} \{\overset{78}{(+)}\ yA^y_+ + \overset{78}{(-)}\ yA^y_-\}\psi + \text{terms with } \tilde{S}^{ab}. \tag{2.2}$$

We notice, after reading also Table I, that the term with the fields $A^y{}_+$, $y = Y, Y'$, contributes only to masses of $d-$quarks (and electrons), while $A^y{}_-$, $y = Y, Y'$, contributes only to masses of $u-$quarks (and neutrinos).

Let us now rearrange in a similar way all the contributions to the mass matrices by introducing $\mathcal{L}_m = \psi^+\gamma^0\gamma^s p_{0s}\psi = \psi\ \gamma^0\{\{\overset{78}{(+)}\ p_{0+} + \overset{78}{(-)}\ p_{0-}\}\psi$, with $p_{0\pm} = (p_7 \mp i\ p_8) - \frac{1}{2}S^{ab}\omega_{ab\pm} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\pm}, \omega_{ab\mp} = \omega_{ab7} \mp i\ \omega_{ab8}, \tilde{\omega}_{ab\pm} = \tilde{\omega}_{ab7} \mp i\ \tilde{\omega}_{ab8}$. We first rearrange the contribution of $\tilde{S}^{ab}$ to the diagonal terms by introducing the appropriate superposition of $\tilde{S}^{ab} : \tilde{\mathcal{N}}_3^{\pm} := \frac{1}{2}(\tilde{\mathcal{S}}^{12} \pm i\tilde{\mathcal{S}}^{03}), \tilde{\tau}^{13} := \frac{1}{2}(\tilde{\mathcal{S}}^{56} - \tilde{\mathcal{S}}^{78}), \tilde{Y} = \tilde{\tau}^{41} + \tilde{\tau}^{21}, \tilde{Y'} = \tilde{\tau}^{41} - \tilde{\tau}^{21}, \tilde{\tau}^{21} := \frac{1}{2}(\tilde{\mathcal{S}}^{56} + \tilde{\mathcal{S}}^{78}), \tilde{\tau}^{41} := -\frac{1}{3}(\tilde{\mathcal{S}}^{910} + \tilde{\mathcal{S}}^{1112} + \tilde{\mathcal{S}}^{1314})$. Taking into account that $\frac{1}{2}S^{st}\omega_{st\pm} = YA^Y{}_\pm + Y'A^{Y'}{}_\pm + \tau^{13}A^{13}{}_\pm, \frac{1}{2}\tilde{S}^{st}\tilde{\omega}_{st\pm} = \tilde{Y}\tilde{A}^{\tilde{Y}}{}_\pm + \tilde{Y'}\tilde{A}^{\tilde{Y'}}{}_\pm + \tilde{\tau}^{13}\tilde{A}^{13}{}_\pm, \frac{1}{2}\tilde{S}^{mn}\tilde{\omega}_{mn\pm} = \tilde{N}^{+3}\tilde{A}^{+3}{}_\pm + \tilde{N}^{-3}\tilde{A}^{-3}{}_\pm$, with the pairs $(m,n) = (0,3),(1,2)$; $(s,t) = (5,6),(7,8)$, belonging to the Cartan sub algebra and $\omega_\pm = \Omega_7 \mp i\Omega_8$, where Ω_7, Ω_8 stay for any of the above fields, correspondingly, we find $A^{13}{}_\pm = \frac{1}{2}(\omega_{56\pm} - \omega_{78\pm}), A^Y{}_\pm = \frac{1}{2}(A^{41}{}_\pm + \frac{1}{2}(\omega_{56\pm} + \omega_{78\pm})), A^{Y'}{}_\pm = \frac{1}{2}(A^{41}{}_\pm - \frac{1}{2}(\omega_{56\pm} + \omega_{78\pm})), \tilde{A}^{13}{}_\pm = \frac{1}{2}(\tilde{\omega}_{56\pm} - \tilde{\omega}_{78\pm}), \tilde{A}^{\tilde{Y}}{}_\pm = \frac{1}{2}(\tilde{A}^{41}{}_\pm + \frac{1}{2}(\tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm})), \tilde{A}^{\tilde{Y'}}{}_\pm = \frac{1}{2}(\tilde{A}^{41}{}_\pm - \frac{1}{2}(\tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm})), \tilde{A}^{\tilde{N}^{+}{}_3}{}_\pm = \frac{1}{2}(\tilde{\omega}_{12\pm} - i\ \tilde{\omega}_{03\pm}), \tilde{A}^{\tilde{N}^{-}{}_3}{}_\pm = \frac{1}{2}(\tilde{\omega}_{12\pm} + i\ \tilde{\omega}_{03\pm})$, where the fields $A^y{}_\pm$, $y = 13, 41, Y, Y'$, and $\tilde{A}^{\tilde{y}}_\pm$, $\tilde{y} = +3, -3, 13, 41, \tilde{y}, \tilde{y'}$, are uniquely expressible with the corresponding spin connection fields.

The operators, which contribute to non diagonal terms in mass matrices, are superposition of $\tilde{S}^{ab}$ and can be written in terms of nilpotents as follows $\stackrel{ab}{(\tilde{k})}\stackrel{cd}{(\tilde{l})}$, with indices (ab) and (cd) which belong to the Cartan sub algebra indices. We may write accordingly

$$\sum_{a,b \text{ not in Cartan}} \frac{1}{2}\, \tilde{s}^{ab}\omega_{\tilde{ab}\pm} = \sum_{(ac),(bd) \text{ in Cartan k,l}} \stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\; \tilde{A}^{kl}{}_{\pm}((ac),(bd)), \tag{2.3}$$

where $k = \pm i$, if $(ac) = (03)$ and $k = \pm 1$ in all other cases, while $l = \pm 1$. The pairs $(ac),(bd)$ run over all the Cartan sub algebra pairs $((03),(12);\ (03),(56)\ ;(03),(78);\ (12),(56);\ (12),(78))$, while k and l run over all possible four combinations. The fields $\tilde{A}^{kl}{}_{\pm}((ac),(bd))$ can then be expressed by $\tilde{\omega}_{ab\pm}$ as follows: $\tilde{A}^{++}{}_{\pm}((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} - \frac{i}{r}\tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} - \frac{1}{r}\tilde{\omega}_{bd\pm}), \tilde{A}^{--}{}_{\pm}((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} + \frac{i}{r}\tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} - \frac{1}{r}\tilde{\omega}_{bd\pm}), \tilde{A}^{-+}{}_{\pm}((ac),(bd) = \frac{i}{2}(\tilde{\omega}_{ac\pm} + \frac{i}{r}\tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} + \frac{1}{r}\tilde{\omega}_{bd\pm}), \tilde{A}^{+-}{}_{\pm}((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} - \frac{i}{r}\tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} + \frac{1}{r}\tilde{\omega}_{bd\pm})$, with $r = i$, if $(ac) = (03)$, otherwise $r = 1$. We simplify the index klin the exponent of fields $\tilde{A}^{kl}{}_{\pm}((ac),(bd))$ to $\pm$.

We can accordingly write the Lagrange density determining the masses of fermions as follows

$$\begin{aligned}
\mathcal{L}_m = \psi^{+}\gamma^0\, \{\stackrel{78}{(+)}\, (p_{+} - \sum_{y=Y,Y'} yA^{y}_{+} - \sum_{\tilde{y}=\tilde{N}^{+}{}_3,\tilde{N}^{-}{}_3,\tilde{\tau}^{13},\tilde{Y},\tilde{Y}'} \tilde{y}\tilde{A}^{\tilde{y}}_{+}\ ) +\\
\stackrel{78}{(-)}\, (p_{-} - \sum_{y=y,y'} yA^{y}_{-} - \sum_{\tilde{y}=\tilde{N}^{+}{}_3,\tilde{N}^{-}{}_3,\tilde{\tau}^{13},\tilde{Y},\tilde{Y}'} \tilde{y}\tilde{A}^{\tilde{y}}_{-}\ ) -\\
\sum_{\{(ac)(bd)\},k,l} \stackrel{78}{(+)}\stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\; \tilde{A}^{kl}_{+}((ab),(cd)) \ - \sum_{\{(ac)(bd)\},k,l} \stackrel{78}{(-)}\stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\; \tilde{A}^{kl}_{-}((ab),(cd))\}\psi,
\end{aligned} \tag{2.4}$$

with all possible pairs $((ac),(bd))$, which are the members of the Cartan sub algebra and $k = \pm i$, if $(ac) = (03)$, otherwise $k = \pm 1$, $l = \pm 1$. We omitted the term with τ^{13}, since being applied on the right handed spinors it contributes zero. The mass matrix, which follows from these Lagrange density is in general not Hermitian.

III. AN EXAMPLE OF MASS MATRICES FOR QUARKS OF FOUR FAMILIES

Let us make, for simplicity, one further assumption, that there are no terms, which would transform $\stackrel{56}{(+)}$ into $\stackrel{56}{[+]}$. We then end with four families of quarks and leptons

$$\begin{aligned}
&I.\ \stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid ...\\
&II.\ \stackrel{03}{[+i]}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid ...\\
&III.\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \mid\mid ...\\
&IV.\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \mid\mid ....
\end{aligned} \tag{3.1}$$

This leads to the mass matrices of the type

	I_R	II_R	III_R	IV_R
I_L	${A^I}_{\mp}$	$\tilde{A}^{++}{}_{\mp}((03),(12))$	$\pm\tilde{A}^{++}{}_{\mp}((03),(78))$	$\mp\tilde{A}^{++}{}_{\mp}((12),(78))$
II_L	$\tilde{A}^{--}{}_{\mp}((03),(12))$	${A^{II}}_{\mp}$	$\pm\tilde{A}^{-+}{}_{\mp}((12),(78))$	$\mp\tilde{A}^{-+}{}_{\mp}((03),(78))$
III_L	$\pm\tilde{A}^{--}{}_{\mp}((03),(78))$	$\mp\tilde{A}^{+-}{}_{\mp}((12),(78))$	${A^{III}}_{\mp}$	$\tilde{A}^{-+}{}_{\mp}((03),(12))$
IV_L	$\pm\tilde{A}^{--}{}_{\mp}((12),(78))$	$\mp\tilde{A}^{+-}{}_{\mp}((03),(78))$	$\tilde{A}^{+-}{}_{\mp}((03),(12))$	${A^{IV}}_{\mp}$

Table II. The mass matrices for four families of quarks and leptons in the approach unifying spins and charges, obtained under some simplifying assumptions. The values ${A^i}_-$, $i = I, II, II, IV$, and $\tilde{A}^{lm}{}_-((ac),(bd))$ determine matrix elements for the u quarks and the neutrinos, the values ${A^i}_+$, $i = I, II, III, IV$, and $\tilde{A}^{lm}{}_+((ac),(bd))$ determine the matrix elements for the d quarks and the electrons. Diagonal matrix elements are different for quarks then for leptons and distinguish also between the u and the d quarks and between the ν and the e leptons. They also differ from family to family. Non diagonal matrix elements distinguish among families and among (u,ν) and (d,e).

The explicit form of the diagonal matrix elements for the above choice of assumptions in terms of ω_{abc} and $\tilde{\omega}_{abc}$ is as follows: ${A^I}_u = -\frac{1}{2}(\omega_{56-} + \omega_{78-} + \frac{1}{3}A^{41}{}_- + i\tilde{\omega}_{03-} + \tilde{\omega}_{12-} + \tilde{\omega}_{56-} + \tilde{\omega}_{78-} + \frac{1}{3}\tilde{A}^{41}{}_-)$,
${A^I}_\nu = -\frac{1}{2}(\omega_{56-} + \omega_{78-} - A^{41}{}_- + i\tilde{\omega}_{03-} + \tilde{\omega}_{12-} + \tilde{\omega}_{56-} + \tilde{\omega}_{78-} + \frac{1}{3}\tilde{A}^{41}{}_-)$,
${A^I}_d = -\frac{1}{2}(-\omega_{56+} - \omega_{78+} + \frac{1}{3}A^{41}{}_+ + i\tilde{\omega}_{03+} + \tilde{\omega}_{12+}\tilde{\omega}_{56+} + \tilde{\omega}_{78+} + \frac{1}{3}\tilde{A}^{41}{}_+)$,
${A^I}_e = -\frac{1}{2}(-\omega_{56+} - \omega_{78+} - A^{41}{}_+ + i\tilde{\omega}_{03+} + \tilde{\omega}_{12+}\tilde{\omega}_{56+} + \tilde{\omega}_{78+} + \frac{1}{3}\tilde{A}^{41}{}_+)$,
${A^{II}}_u = {A^i}_u + (i\tilde{\omega}_{03-} + \tilde{\omega}_{12-})$, ${A^{II}}_\nu = {A^i}_\nu + (i\tilde{\omega}_{03-} + \tilde{\omega}_{12-})$,
${A^{II}}_d = {A^i}_d + (i\tilde{\omega}_{03+} + \tilde{\omega}_{12+})$, ${A^{II}}_e = {A^i}_e + (i\tilde{\omega}_{03+} + \tilde{\omega}_{12+})$,
${A^{III}}_u = {A^i}_u + (i\tilde{\omega}_{03-} + \tilde{\omega}_{78-})$, ${A^{III}}_\nu = {A^i}_\nu + (i\tilde{\omega}_{03-} + \tilde{\omega}_{78-})$,
${A^{III}}_d = {A^i}_d + (i\tilde{\omega}_{03+} + \tilde{\omega}_{78+})$, ${A^{III}}_e = {A^i}_e + (i\tilde{\omega}_{03+} + \tilde{\omega}_{78+})$,
${A^{IV}}_u = {A^i}_u + (\tilde{\omega}_{12-} + \tilde{\omega}_{78-})$, ${A^{IV}}_\nu = {A^i}_\nu + (\tilde{\omega}_{12-} + \tilde{\omega}_{78-})$,
${A^{IV}}_d = {A^i}_d + (\tilde{\omega}_{12+} + \tilde{\omega}_{78+})$, ${A^{IV}}_e = {A^i}_e + (\tilde{\omega}_{12+} + \tilde{\omega}_{78+})$. The explicit form of nondiagonal matrix elements is as follows $\tilde{A}^{++}{}_\pm((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} - \frac{i}{r}\tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} - \frac{1}{r}\tilde{\omega}_{bd\pm})$, $\tilde{A}^{--}{}_\pm((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} + \frac{i}{r}\tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} - \frac{1}{r}\tilde{\omega}_{bd\pm})$, $\tilde{A}^{-+}{}_\pm((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} + \frac{i}{r}\tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} + \frac{1}{r}\tilde{\omega}_{bd\pm})$, $\tilde{A}^{+-}{}_\pm((ac),(bd)) = \frac{i}{2}(\tilde{\omega}_{ac\pm} - \frac{i}{r}\tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} + \frac{1}{r}\tilde{\omega}_{bd\pm})$.

IV. DISCUSSIONS AND CONCLUSION

In this paper we derive from our approach unifying spins and charges the mass matrices for the quarks and the leptons. The approach assumes that a Weyl spinor of a chosen handedness carries in $d(=1+13)-$ dimensional space nothing but two kinds of spin degrees of freedom. One kind belongs to the Poincaré group in $d = 1 + 13$, another kind generates families. Spinors interact with only the gravitational fields, manifested by the vielbeins and spin connections, the gauge fields of the momentum p_α and the two kinds of the generators of the Lorentz group Sab and $\tilde{S}^{ab}$, respectively. To come to the Yukawa couplings, postulated by the Standard model, we made several further assumptions about the way of breaking symmetries down to observable ones, that is from $SO(1,13)$ to $S(1,7) \times SU(3) \times U(1)$ (which may appear in steps and must lead to massless spinors of a particular handedness, chirally coupled to the $U(1)$ and $SU(3)$ gauged fields) and then further to $SO(1,3) \times SU(2) \times SU(3) \times U(1)$.

Our starting Weyl spinor representation of a chosen handedness manifests, if analysed in terms of the subgroups $SO(1,3), SU(3), SU(2)$ and two $U(1)'$s of the group $SO(1,13)$, the spin and all the charges of one family of quarks and leptons.

We use our technique[13,14] to present spinor representations in a transparent way so that one easily sees how (and why) the part of the covariant derivative of a spinor in $d = 1 + 13$ manifests in $d = 1 + 3$ as Yukawa couplings of the Standard model.

We predict an even number of families. Making one further assumption, we predict four families at low energies, rather than three and we present the mass matrices as the function of the spin connection fields for the quarks and the leptons on the classical level. From these matrix elements we calculated, by fitting the experimental data the masses of the two fourth quarks, predicting $m_{u_4} := 259GeV - 295GeV$ and $m_{d_4} := 180GeV - 390GeV$, while the mixing matrix $S^{+}{}_{d}S_u$ is

$$V_{min} \begin{pmatrix} 0.974 & 0.220 & 0.004 & 0.040 \\ 0.224 & 0.974 & 0.040 & 0.004 \\ 0.004 & 0.040 & 0.930 & 0.40 \\ 0.040 & 0.004 & 0.400 & 0.920 \end{pmatrix}.$$

These results are preliminary and ought to be tested.

REFERENCES

[1] N. S. Mankoč Borštnik, *Phys. Lett.* **B 292**, 25-29 (1992).
[2] N. S. Mankoč Borštnik, *J. Math. Phys.* **34**, 3731-3745 (1993).
[3] N. Mankoč Borštnik, *J. Math. Phys.* **36**, 1593-1601(1994),
[4] N. S. Mankoč Borštnik, *Modern Phys. Lett. A* **10**, 587-595 (1995),
[5] N. S. Mankoč Borštnik and S. Fajfer, *N. Cimento* **112B**, 1637-1665(1997).
[6] A. Borštnik, N. S. Mankoč Borštnik, hep-ph/9905357.
[7] N. S. Mankoč Borštnik, H. B. Nielsen, *Phys. Rev.* **62** (04010-14) (2000),
[8] N. S. Mankoč Borštnik, *Int. J. Theor. Phys.* **40**, 315-337 (2001).
[9] A. Borštnik, N. S. Mankoč Borštnik, hep-ph/0301029.
[10] A. Borštnik, N. S. Mankoč Borštnik, hep-ph/0401043, hep-ph/0401055, hep-ph/0301029.
[11] E. Witten, Nucl. Phys. B **186** 412 (1981); Princeton Technical Rep. PRINT -83-1056, October 1983.
[12] N. S. Mankoč Borštnik, H. B. Nielsen, hep-th/0509101.
[13] N. S. Mankoč Borštnik, H. B. Nielsen, J. of Math. Phys. **43**, 5782-5803, hep-th/0111257.
[14] N. S. Mankoč Borštnik, H. B. Nielsen, *J. of Math. Phys.* **44** (2003) 4817-4827, hep-th/0303224.
[15] N. S. Mankoč Borštnik, H.B. Nielsen, hep-ph/0301029.
[16] V.A. Novikov, L.B. Okun,A.N. Royanov, M.I. Vysotsky, "Extra generations and discrepancies of electroweak precision data", hep-ph/0111028.
[17] A. Kleppe, D. Lukman, N.S. Mankoč Borštnik, hep-ph/0301029.
[18] M.Breskvar, J. Mravlje, N.S. Mankoč Borštnik, hep-ph/0412208.