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Complete one–loop: to stop and sbottom decays into Z and $W^{\pm}$ bosons

Abdesslam Arhrib[1,2] and Rachid Benbrik[2]

[1] *Département de Mathématiques, Faculté des Sciences et Techniques, B.P 416 Tanger, Morocco.*
[2] *LPHEA, Département de Physique, Faculté des Sciences-Semlalia, B.P. 2390 Marrakech, Morocco.*

Abstract

The complete one–loop radiative corrections to third generation scalar fermions into gauge bosons Z and $W^{\pm}$ is considered. We focus on $\widetilde{f}_2 \to Z\widetilde{f}_1$ and $\widetilde{f}_i \to W^{\pm}\widetilde{f}'_1$, $f, f' = t, b$. We include both SUSY-QCD, QED and full electroweak corrections. It is found that the electroweak corrections can be of the same order as the SUSY-QCD corrections. The two sets of corrections interfere destructively in some region of parameter space. The full one loop correction can reach 10% in some SUGRA scenario, while in model independent analysis like general MSSM, the one loop correction can reach 20% for large $\tan\beta$ and large trilinear soft breaking terms A_b.

I. INTRODUCTION

Supersymmetric theories predict the existence of scalar partners to all known quarks and leptons[1]. In Grand unified SUSY models, the third generation of scalar fermions, $\widetilde{t}, \widetilde{b}, \widetilde{\tau}$, gets a special status; due to the influence of Yukawa-coupling evolution, the light scalar fermions of the third generation are expected to be lighter than the scalar fermions of the first and second generations. For the same reason, the splitting between the physical masses of the third generation may be large enough to allow the opening of the decay channels like : $\widetilde{f}_2 \to \widetilde{f}_1 V$ and/or $\widetilde{f}_2 \to \widetilde{f}_1 \Phi$, where V is a gauge boson and Φ is a scalar boson.

Until now there is no direct evidence for SUSY particles, and under some assumptions on their decay rates, one can only set lower limits on their masses[2]. It is expected that the next generation of e^+e^- machines and/or hadron colliders (LHC and Tevatron) could establish the first evidence for the existence of SUSY particles. Typically, Scalar quarks can be produced copiously both at hadron and lepton colliders. They can in principle be discovered at future hadron colliders (LHC) up to masses in the 1-2 TeV range while sleptons would become invisible to LHC if heavier than $\sim$ 250 GeV or so[3], due to their weak coupling and a prominent background.

If SUSY particles would be detected at hadron colliders, their properties can be studied with high accuracy at a high-energy linear e^+e^- collider[4]. It is thus mandatory to incorporate effects beyond leading order into the theoretical predictions, both for production and decay rate, in order to match the experimental accuracy.
In this spirit, the next-to-leading order corrections to squark-pair production at proton colliders have

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been studied theoretically in[5] and found to increase the cross section. For e^+e^- machines, scalar-fermion production has been addressed in several studies and shown to be promising for precision analysis of sfermion properties with mass and mixing-angle reconstructions[4]. SUSY-QCD corrections to squark-pair production at e^+e^- annihilation were shown, a decade ago, to be large[6,7]. Recently, the full one-loop radiative corrections to the production of scalar muons, scalar electrons (near threshold), and third generation scalar fermions $\tilde{t}, \tilde{b}, \tilde{\tau}$[9,8] have been addressed. For squark pair production at e^+e^-, the leading and subleading electroweak Sudakov logarithms were investigated[10] and found to be large at high energy. Similar studies have been carried out for the decays of SUSY particles. In particular, the QCD corrections to scalar quark decay into quarks plus charginos or neutralinos have been studied in[11], while the full one loop analysis has been addressed in[12] and found to have important impact on the partial decay widths of scalar fermions. In Ref.[13], the QCD corrections to the decays of heavy scalar quarks into light scalar quarks and Higgs bosons are found to be of the order 10 → 20 %.

Obviously, most of the studies concentrated on the production and decay of light states $\tilde{t}_1, \tilde{b}_1$ and $\tilde{\tau}_1$, while heavier states received less attention[8,12–14]. These heavy states can be produced both at LHC and/or at the future e^+e^- linear colliders. The decay of the heavier states third generation scalar fermions is more complicated than the light one. One can basically have four set of two-body decays:
i) Strong decay for stop and sbottom $\tilde{t}_2 \to t\tilde{g}$, $\tilde{b}_2 \to b\tilde{g}$: if these decay are kinematically open they are the dominant one.
ii) decay to chargino and neutralino : $\tilde{f}_2 \to f\tilde{\chi}_i^0$, $\tilde{f}_2 \to f'\tilde{\chi}_i^+$.
If the splitting between light and heavy third generation scalar fermions is large enough we may have the following decays:
iii) $\tilde{f}_2 \to \tilde{f}_1\Phi^0$, $\Phi^0 = h^0, H^0, A^0$, and $\tilde{f}_2 \to \tilde{f'}_1 H^\pm$.
iv) $\tilde{f}_2 \to \tilde{f}_1 Z^0$ and $\tilde{f}_2 \to \tilde{f'}_1 W^\pm$.
It has been shown in[15] that the decay modes $\tilde{f}_2 \to \tilde{f}_1 Z^0$ and $\tilde{f}_i \to \tilde{f'}_j W$, if open and under some assumptions, may be the dominant one. Ref.[15] also evaluate the gueniun SUSY-QCD corrections and found them to be of the order -5% → -10%.
Note also that in several benchmarks scenario for SUSY searches, the bosonic decay of $\tilde{t}_i$ and $\tilde{b}_i$ may be the dominant[16]. For example, in SPS5 scenario the dominant bosonic decay have the following branching ratios[17]: $Br(\tilde{b}_1 \to W^-\tilde{t}_1) = 81\%$, $Br(\tilde{b}_2 \to W^-\tilde{t}_1) = 64\%$ and $Br(\tilde{t}_2 \to Z^0\tilde{t}_1) = 61\%$. While in SPS1 scenario, we have: $Br(\tilde{b}_2 \to W^-\tilde{t}_1) = 34\%$ and $Br(\tilde{t}_2 \to Z^0\tilde{t}_1) = 23\%$.

In this study we review our work on the branching ratios of squarks decays. In order to show how the branching ratios of the various decay modes depend on the SUSY parameters, we will first summarize the tree-level results.Then we will take into account SUSY - full corrections for the decay branching rations into gauge bosons Z and $W^\pm$.
We will show that in most cases the SUSY - full corrections are significant and need to be included.

II. TREE–LEVEL FORMULAE

The interaction of the neutral gauge bosons γ and Z with the sfermion mass eigenstates is described by the Lagrangian

$$\begin{aligned}\mathcal{L} = -ieA^\mu \sum_{i=1,2} Q_f \tilde{f}_i^* \overleftrightarrow{\partial}_\mu \tilde{f}_i - ig_s G_a^\mu \sum_{i=1,2} T^a \tilde{f}_i^* \overleftrightarrow{\partial}_\mu \tilde{f}_i + \\ iZ^\mu \sum_{i,j=1,2} g_{Z\tilde{f}_i\tilde{f}_j} \tilde{f}_i^* \overleftrightarrow{\partial}_\mu \tilde{f}_j + iW^\mu \sum_{i,j=1,2} g_{W\tilde{f}_i\tilde{f}'_j} \tilde{f}_i^* \overleftrightarrow{\partial}_\mu \tilde{f'}_j\end{aligned} \tag{2.1}$$

with

$$\begin{aligned}g_{Z\tilde{f}_i\tilde{f}_j} &= -\frac{e}{s_W c_W}\{(I_3^f - Q_f s_W^2) R_{j1}^{\tilde{f}} R_{i1}^{\tilde{f}} - Q_f s_W^2 R_{j2}^{\tilde{f}} R_{i2}^{\tilde{f}}\} \\ g_{W\tilde{f}_i\tilde{f'}_j} &= -\frac{e}{\sqrt{2}s_W} R_{i1}^f R_{j1}^{f'}\end{aligned} \tag{2.2}$$

The tree–level decay width can thus be written as:

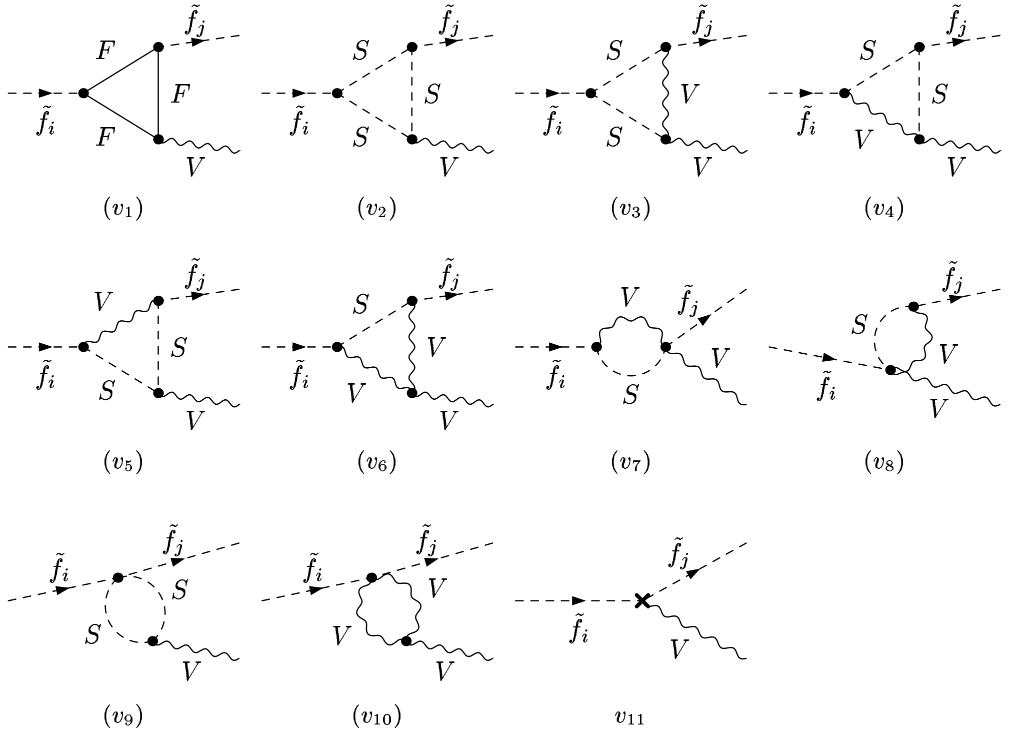
$$\Gamma^0(\tilde{q}_i^\alpha \to \tilde{q}_j^\beta V) = \frac{(g_{V\tilde{f}_i\tilde{f}_j})^2\, \kappa^3(m_i^2, m_j^2, m_V^2)}{16\pi\, m_V^2\, m_i^3}, \qquad (2.3)$$

with $\kappa(x,y,z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{1/2}$.

III. RADIATIVE CORRECTIONS

A. Scalar fermions decay into gauge bosons at one loop

The Feynman diagrams for the one-loop virtual contributions are generically displayed in (Fig. 1)$(v_{1,\ldots,10})$. These diagrams are to be supplemented by the external self-energy contributions for gauge bosons and scalar fermions $\tilde{f}_{i,j}$ (Fig. 2), which are part of the counter-term for vertices (Fig. 1)(v_{11}), to be added according to renormalization. In the generic notation, V, S, F denote all insertions of vector, scalar, and fermionic states. At one loop level, transitions between gauge bosons and scalar bosons like $W^\pm$-$H^\pm$, $W^\pm$-$G^\pm$, Z^0-A^0, Z^0-G^0 are present.



(v_1) (v_2) (v_3) (v_4)

(v_5) (v_6) (v_7) (v_8)

(v_9) (v_{10}) v_{11}

FIG. 1. Generic vertex contributions to $\tilde{f}_i \to \tilde{f}_j^* V$

The full set of Feynman diagrams are generated and evaluated using the packages FeynArts and FormCalc[18]. We have also used LoopTools and FF[19] in the numerical analysis.

We have evaluated the one-loop amplitudes in the 't Hooft–Feynman gauge. The one-loop amplitudes are ultraviolet (UV) and infrared (IR) divergent. The UV singularities are treated by dimensional reduction and are compensated

In case of $\tilde{f}_2 \to Z\tilde{f}_1$ decay, diagrams like (Fig. 1)(v_5) with $V = \gamma$ or $V =$ *gluon* and diagram (Fig. 1)(v_{11}) are IR divergent. In (Fig. 1)(v_{11}) the IR divergence comes from the wave function renormalization of the scalar fermions. While for $\tilde{f}_2 \to W\tilde{f'}_1$ decay, diagrams like (Fig. 1)$(v_{3,\ldots,6})$ and (Fig. 1)(v_{11}) $V = \gamma$ or $V =$ *gluon* are IR divergent, for an IR-finite decay width we have to add the contribution from real-photon and real-gluon emission, $\tilde{f}_i \to \tilde{f}_j^* V\gamma$ and $\tilde{f}_i \to \tilde{f}_j^* Vg$.

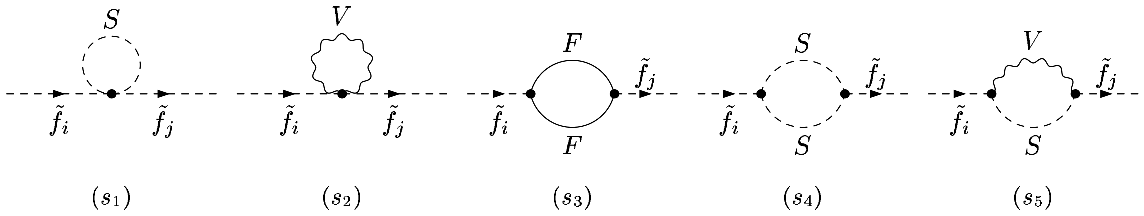


FIG. 2. Generic Feynman diagrams for Scalar fermions self-energies $\widetilde{f}_i \to \widetilde{f}_j$

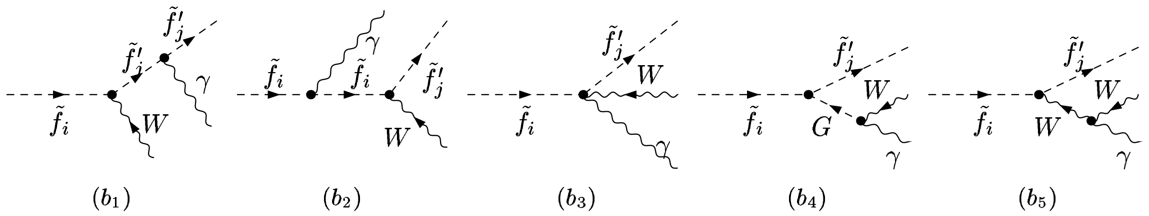


FIG. 3. Feynman diagrams for real photon and gluon emission for in the final state of $\widetilde{f}_i \to \widetilde{f}_j V$

In order to cancel the infrared divergence coming from virtual gluon, the real corrections with an additional gluon in the final state need also to be included. Feynman diagrams contributing to $\delta\Gamma^{br}_g = \Gamma(\widetilde{q}_i \to \widetilde{q}_j\, V\, g)$ are drawn in (Fig.3)(b_1, b_2, b_3). The magnitude of SUSY radiative corrections can be described by the relative correction which we define as:

$$\Delta = \frac{\Gamma^{1-\text{loop}}(\widetilde{f}_i \to \widetilde{f}_j V) - \Gamma^{\text{tree}}(\widetilde{f}_i \to \widetilde{f}_j V)}{\Gamma^{\text{tree}}(\widetilde{f}_i \to \widetilde{f}_j V)} \tag{3.1}$$

As in the case of gluon, the infrared divergence coming from virtual photon cancels out by including real (soft and hard) photon emission in the final state. The diagrams contributing to real bremsstrahlung of $\widetilde{f}_i \to \widetilde{f}_j\, Z$ are depicted in (Fig.3)(b_1, b_2, b_3).

In case of $\widetilde{f}_i \to \widetilde{f}_j\, Z\, \gamma$, the width can be deduced from the gluon bremsstrahlung. Recently, there have been several developments in the renormalization of MSSM. Several schemes are available[21−23]. Here, we follow the strategy of[12] by introducing counter-terms for the physical parameters, i.e. for masses and mixing angles, and perform field renormalization in a way that residues of renormalized propagators can be kept at unity.

We will adopt throughout, the on–shell renormalization scheme of Refs.[20] for SM parameters and fields.

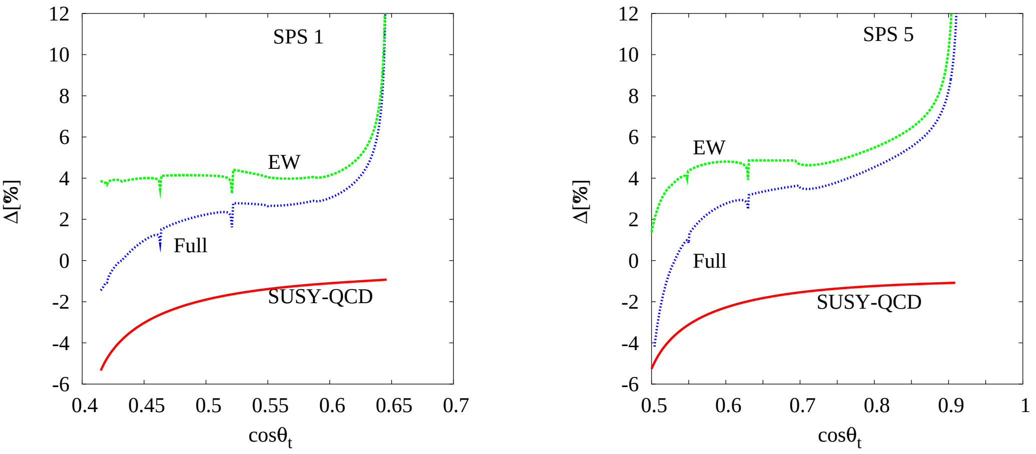


FIG. 4. Relative correction (electroweak EW, SUSY-QCD and full) to $\tilde{b}_2 \to \tilde{t}_1 W$ as function of $\cos\theta_t$ in SPS1 (left) and SPS5 (right)

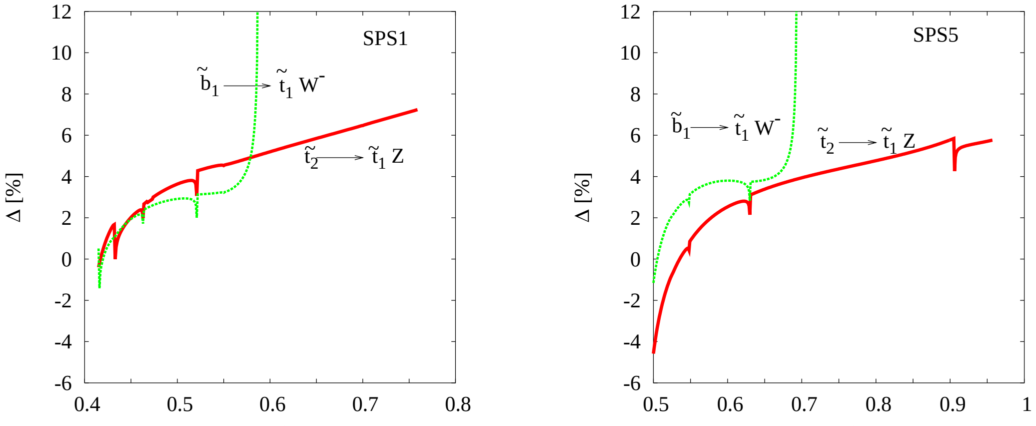


FIG. 5. Relative correction to $\tilde{b}_1 \to \tilde{t}_1 W$ and $\tilde{t}_2 \to \tilde{t}_1 Z$ as function of $\cos\theta_t$ in SPS1 (left) and SPS5 (right)

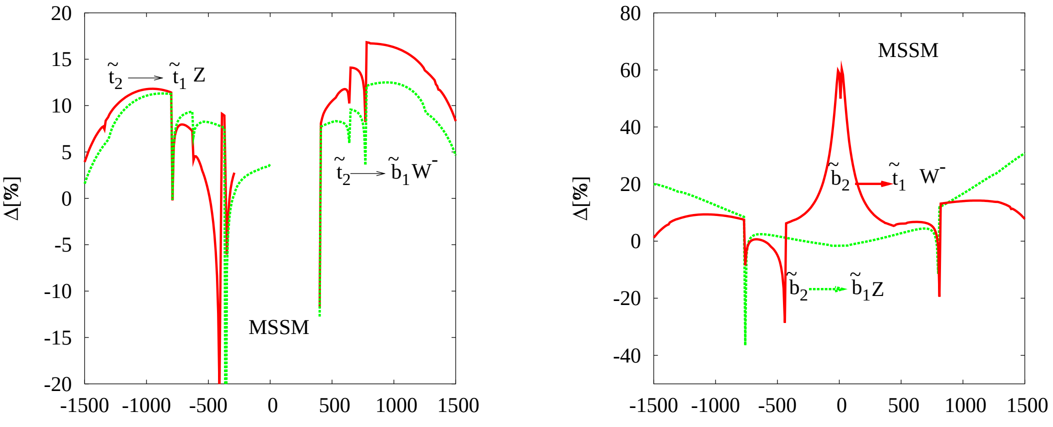


FIG. 6. Relative correction to $\tilde{t}_2 \to \tilde{b}_1 W$,$\tilde{t}_2 \to \tilde{t}_1 Z$ (left) and $\tilde{b}_2 \to \tilde{b}_1 Z$,$\tilde{b}_2 \to \tilde{t}_1 W$ (right) as function of $A_t = A_b$ in general MSSM for $\mu = 500$ GeV, $M_2 = 130$ GeV, $M_A = 200$ GeV and $\tan\beta = 60$

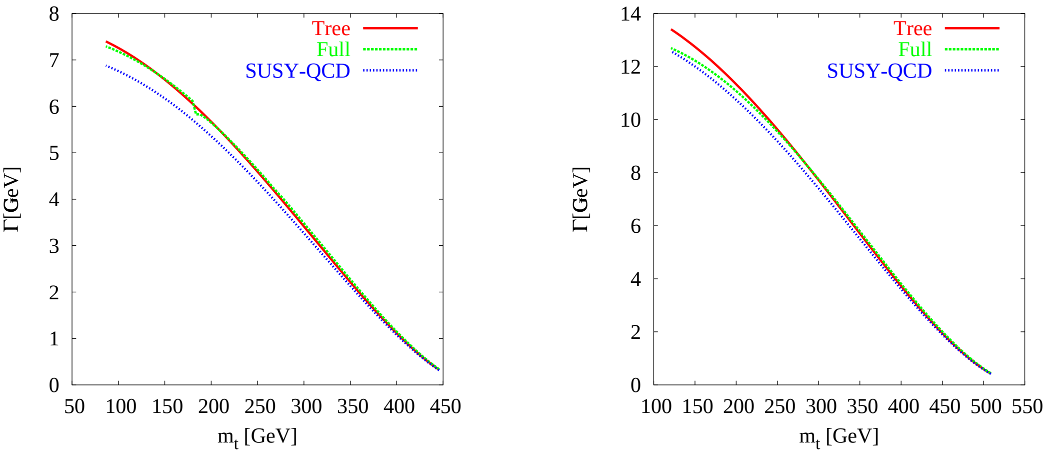


FIG. 7. Tree and one loop decay width of $\tilde{t}_2 \to \tilde{t}_1 Z$ as function of $m_{\tilde{t}_1}$

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