



Some Remarks on Tree-Level Vacuum Stability in Two Higgs Doublet Models

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Abstract

It is proved that the minimum of a general two Higgs doublet model's potential is stable at tree level. A relation between stability and flavor changing neutral currents at tree level is shown.

I. INTRODUCTION

One of the most straightforward ways to extend the Standard Model of the weak interactions (SM) is to add a second Higgs doublet to the scalar sector. This type of models is known in the literature as two Higgs doublet models (2HDM). They present a richer phenomenology due to the appearance of charged and also pseudo-scalar Higgs. However, maybe one of the main reasons of interest in this class of models is the possibility of having spontaneous CP violation¹, thus helping to solve the baryogenesis problem² (for a review, see³). These models have a very large number of independent parameters. The most general 2HDM has 14 real parameters although, with a particular choice of basis, one can reduce this number to 11 independent parameters (see, for instance,⁴). There are some bounds on the parameters in models derived from the general one by imposing a Z_2 or a $U(1)$ symmetry. However, besides some very weak experimental and theoretical bounds, very little is known about their allowed values, especially in this most general case. The same is true for Supersymmetric models, where the parameter space is generally larger. One idea that has been applied to Supersymmetric theories to restrict their allowed parameter space is to use charge and colour breaking (CCB) bounds. If a given combination of parameters causes the appearance in the potential of a minimum where charged/coloured fields have vacuum expectation values (vevs), then that combination should be rejected. This appealing idea was introduced by Frère *et al*⁵ and applied, in numerous papers, to several supersymmetric theories⁶. Phenomenological analysis of supersymmetric Higgs masses use this tool to increase the models' predictive power⁷. It is therefore of interest to apply similar techniques to the 2HDM and try to limit its parameter space. The scalars of this theory have no colour quantum numbers but there are charged fields so charge breaking (CB) extrema are in principle possible. It was shown in ref.⁸ that to assure that there were no stationary points corresponding to charge or CP spontaneous breaking, one had to restrict the parameter space to 7 independent parameters. This led to two independent potentials, stable under renormalisation, as they were protected by a Z_2 or a global $U(1)$ symmetries (that can be softly broken). It is interesting to stress that these are the usual symmetries introduced to prevent flavor changing neutral currents. We will come back to this point later.

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II. TREE-LEVEL STABILITY

This section follows very closely the work done in ^{9,7}. There are two scalar Higgs doublets in the theory, Φ_1 and Φ_2 , both having hypercharge $Y = 1$, i.e.,

$$\Phi_1 = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_5 + i\varphi_7 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \varphi_3 + i\varphi_4 \\ \varphi_6 + i\varphi_8 \end{pmatrix}. \quad (2.1)$$

The numbering of the real scalar φ fields is chosen for convenience in writing the mass matrices for the scalar particles. Our basis is obtained by first writing down the four $SU(2)_W \times U(1)_Y$ invariants one can construct from these two doublets, namely $x_1 \equiv |\Phi_1|^2$, $x_2 \equiv |\Phi_2|^2$, $x_3 \equiv \text{Re}(\Phi_1^\dagger \Phi_2)$ and $x_4 \equiv \text{Im}(\Phi_1^\dagger \Phi_2)$. Notice that under a CP transformation ($\Phi_1 \rightarrow \Phi_1^*$, $\Phi_2 \rightarrow \Phi_2^*$) the invariants x_1 , x_2 and x_3 remain the same but x_4 changes sign. It is now a simple matter to write down the most general 2HDM model, i.e.,

$$V = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{44} x_4^2 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{34} x_3 x_4, \quad (2.2)$$

where the a_i parameters have dimensions of mass squared and the b_{ij} parameters are dimensionless. The terms linear in x_4 are those that break CP explicitly, and eliminating them we are left with the CP preserving potential with 10 parameters that we have used in reference⁹. Notice that this potential has only 9 independent parameters due to basis invariance (see⁴ for details). For convenience we introduce a new notation, namely a vector A and a square matrix B , given by

$$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}, \quad B = \begin{bmatrix} 2b_{11} & b_{12} & b_{13} & b_{14} \\ b_{12} & 2b_{22} & b_{23} & b_{24} \\ b_{13} & b_{23} & 2b_{33} & b_{34} \\ b_{14} & b_{24} & b_{34} & 2b_{44} \end{bmatrix}. \quad (2.3)$$

Defining the vector $X = (x_1, x_2, x_3, x_4)$, we can rewrite the potential (2.2) as

$$V = A^T X + \frac{1}{2} X^T B X. \quad (2.4)$$

It is a well known fact that the 2HDM potential has only three types of possible minima^{1,3}. With our conventions we can define a charge breaking (CB) minimum configuration as

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1' \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \alpha \\ v_2' \end{pmatrix}. \quad (2.5)$$

The vev α breaks the $U(1)_{em}$ symmetry and so gives a mass to the photon. In the second type of minima only neutral fields have vevs, and there are two different possibilities which we define as

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad (2.6)$$

and

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1'' + i\delta \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} 0 \\ v_2'' \end{pmatrix}. \quad (2.7)$$

We call the first the N_1 minimum, and the second the N_2 minimum. When the model is reduced to the explicit CP preserving potential, N_1 is the CP preserving minimum and N_2 is the one that spontaneously breaks CP. For the potential where CP is explicitly broken from the start, there is no physical distinction between the N_1 and N_2 minima.

The demonstration that if one of the normal minima exists, it is certainly deeper than the charge breaking one was done in^{9,7}. We will skip here some non-essential intermediate steps and refer the reader to these articles. Let V' be a vector with components $V_i' = \partial V / \partial x_i$, evaluated at the N_1 minimum. At

N_1 the non-zero vevs are $\varphi_5 = v_1$ and $\varphi_6 = v_2$, so that $x_1 = v_1^2$, $x_2 = v_2^2$, $x_3 = v_1 v_2$ and $x_4 = 0$. We define the vector X_{N_1} as the value of X at this minimum, and it is a trivial matter to demonstrate that the value of the potential at the minimum is given by

$$V_{N_1} = \frac{1}{2} A^T X_{N_1} = -\frac{1}{2} X_{N_1}^T B X_{N_1} . \tag{2.8}$$

Further, we can write down the non-trivial stationarity conditions, which are

$$\begin{aligned} \frac{\partial V}{\partial v_1} = 0 &\Leftrightarrow V'_1 \frac{\partial x_1}{\partial v_1} + V'_3 \frac{\partial x_3}{\partial v_1} = 0 \Leftrightarrow V'_1 = \left(-\frac{V'_3}{2v_1 v_2}\right) v_2^2 \\ \frac{\partial V}{\partial v_2} = 0 &\Leftrightarrow V'_2 \frac{\partial x_2}{\partial v_2} + V'_3 \frac{\partial x_3}{\partial v_2} = 0 \Leftrightarrow V'_2 = \left(-\frac{V'_3}{2v_1 v_2}\right) v_1^2 \\ \frac{\partial V}{\partial \varphi_7} = 0 &\Leftrightarrow V'_4 \frac{\partial x_4}{\partial \varphi_7} = 0 \Leftrightarrow V'_4 = 0 . \end{aligned} \tag{2.9}$$

From eq. (2.8) we see that $V' = A + B X_{N_1}$ and from the equations above we can obtain

$$V' = \begin{bmatrix} V'_1 \\ V'_2 \\ V'_3 \\ V'_4 \end{bmatrix} = -\frac{V'_3}{2v_1 v_2} \begin{bmatrix} v_2^2 \\ v_1^2 \\ -2v_1 v_2 \\ 0 \end{bmatrix} . \tag{2.10}$$

Written in this form we see, plainly, that V'_1 and V'_2 have the same sign. Now, in reference⁹ we have obtained general expressions for the mass matrices of the theory's scalar particles. In particular we have shown that $M_{H^\pm}^2 = V'_1 + V'_2 = -V'_3 v^2 / (2v_1 v_2)$, with $v^2 = v_1^2 + v_2^2$. If N_1 is a minimum then all of the squared scalar masses are positive and so this quantity is positive. Another consequence of the minimization conditions is that we obtain $X_{N_1}^T V' = 0$.

Regarding the CB stationary point, the fields that have non-zero vevs are now $\varphi_5 = v'_1$, $\varphi_6 = v'_2$ and $\varphi_3 = \alpha$. We define the vector Y to be equal to the vector X evaluated at this stationary point, that is, Y has components $Y = (v_1'^2, v_2'^2 + \alpha^2, v_1' v_2', 0)$. The stationarity conditions now give

$$\begin{aligned} \frac{\partial V}{\partial v_1'} = 0 &\Leftrightarrow V'_1 \frac{\partial x_1}{\partial v_1'} + V'_3 \frac{\partial x_3}{\partial v_1'} = 0 \Leftrightarrow V'_1 = \left(-\frac{V'_3}{2v_1' v_2'}\right) v_2'^2 \\ \frac{\partial V}{\partial v_2'} = 0 &\Leftrightarrow V'_2 \frac{\partial x_2}{\partial v_2'} + V'_3 \frac{\partial x_3}{\partial v_2'} = 0 \Leftrightarrow V'_2 = \left(-\frac{V'_3}{2v_1' v_2'}\right) v_1'^2 \\ \frac{\partial V}{\partial \alpha} = 0 &\Leftrightarrow V'_2 \frac{\partial x_2}{\partial \alpha} = 0 \Leftrightarrow V'_2 = 0 \\ \frac{\partial V}{\partial \varphi_7} = 0 &\Leftrightarrow V'_4 \frac{\partial x_4}{\partial \varphi_7} = 0 \Leftrightarrow V'_4 = 0 . \end{aligned} \tag{2.11}$$

We thus obtain, for the CB stationary point, $V'_i = 0$. The equation that determines Y is simply $A + B Y = 0$, which implies that, even for this more complex potential, the CB stationary point, if it exists, is unique. The value of the potential at this charge breaking stationary point is given by

$$V_{CB} = \frac{1}{2} A^T Y = -\frac{1}{2} Y^T B Y . \tag{2.12}$$

Remembering that $X_{N_1}^T V' = 0$ we obtain, from $V' = A + B X_{N_1}$ and $A + B Y = 0$, that

$$X_{N_1}^T B Y = X_{N_1}^T B X_{N_1} = -2 V_{N_1} . \tag{2.13}$$

We can calculate the quantity $Y^T V'$, which is easily seen to be given by

$$Y^T V' = -Y^T B Y + Y^T B X_{N_1} . \tag{2.14}$$

But, from eq. (2.12), it follows that $Y^T B Y = -2 V_{CB}$ and eq. (2.13) and the fact that the matrix B is symmetric gives $Y^T B X_{N_1} = -2 V_{N_1}$. Therefore, we reach the conclusion that

$$V_{CB} - V_{N_1} = \frac{1}{2} Y^T V' = \frac{M_{H^\pm}^2}{2 v^2} [(v'_1 v_2 - v'_2 v_1)^2 + \alpha^2 v_1^2] \quad (2.15)$$

Then, it is clear that, if N_1 is a minimum of the theory, all of its squared masses will be positive, and therefore we will have $V_{CB} - V_{N_1} > 0$, which implies that the charge breaking stationary point, when it exists, is always located above the N_1 minimum. Furthermore, it is easy to obtain the equality $Y = X - B^{-1} V'$, so that $Y^T V'$ becomes equal to $-V'^T B^{-1} V'$. In ref.⁹ we demonstrated that the matrix B determines the nature of the CB stationary point. The equality we have just obtained demonstrates that the matrix B is *not* positive definite. For reasons explained in⁹ it cannot be negative definite (which arises from requiring that the potential we are working with is bounded from below), which implies that B is neither positive nor negative definite. As a result, the CB stationary point is a saddle point.

Now we turn our attention to the N_2 minimum. *A priori* there is no reason why the 2HDM potential cannot have, simultaneously, both “normal” minima, so the question arises, can the potential be in an N_2 minimum that is not deeper than a CB stationary point? The answer is no, and the demonstration follows very closely the one we just concluded. For the N_2 minimum, the fields that have non-zero vevs are $\varphi_5 = v''_1, \varphi_6 = v''_2$ and $\varphi_7 = \delta$, so that the X vector is now given by $X_{N_2} = (v''_1{}^2 + \delta^2, v''_2{}^2, v''_1 v''_2, -v''_2 \delta)$. Solving the stationarity conditions as before, we find that the vector $V' = A + B X_{N_2}$, at this minimum, is given by

$$V'_{N_2} = \begin{bmatrix} V'_1 \\ V'_2 \\ V'_3 \\ V'_4 \end{bmatrix} = -\frac{(V'_3)_{N_2}}{2v''_1 v''_2} \begin{bmatrix} x_2 \\ x_1 \\ -2x_3 \\ -2x_4 \end{bmatrix}, \quad (2.16)$$

and in fact this final expression also applies to the vector V' , evaluated at the N_1 minimum. We still have $X_{N_2}^T V'_{N_2} = 0$ and $-(V'_3)_{N_2} / (2v''_1 v''_2) = (M_{H^\pm}^2 / v^2)_{N_2}$. In this expression the charged scalar mass is the non-zero eigenvalue of the charged mass matrix at the N_2 minimum and v^2 is now given by $v^2 = v''_1{}^2 + v''_2{}^2 + \delta^2$. We are therefore in the exact conditions of the N_1 minimum and conclude, likewise, that

$$V_{CB} - V_{N_2} = \frac{1}{2} Y^T V' = \left(\frac{M_{H^\pm}^2}{2 v^2} \right)_{N_2} [(v''_1 v_2 - v''_2 v_1)^2 + \alpha^2 (v_1^2 + \delta^2) + \delta^2 v_2^2] > 0. \quad (2.17)$$

Again, the charge breaking stationary point lies above the normal minimum, and again it is a saddle point, for the same reasons we have explained before. Unfortunately we cannot apply this procedure to determine whether one of the minima N_1 or N_2 is deeper than the other. If one follows the steps we have outlined, one is left with

$$V_{N_2} - V_{N_1} = \frac{1}{2} \left[\left(\frac{M_{H^\pm}^2}{v^2} \right)_{N_1} - \left(\frac{M_{H^\pm}^2}{v^2} \right)_{N_2} \right] [(v''_1 v_2 - v''_2 v_1)^2 + \delta^2 v_2^2] \quad (2.18)$$

If both N_1 and N_2 are minima - and there does not seem to be anything preventing it - the terms proportional to $M_{H^\pm}^2$ will be positive, but it seems impossible to tell which one is the largest.

For the special case of a potential without explicit CP breaking, the N_2 stationary point is the one with spontaneous CP breaking. The N_1 stationary point preserves both charge and CP and it is what we called, in ref.⁹, the normal minimum. In that reference we calculated the mass matrices for the several minima possible and showed that $(M_{H^\pm}^2 / v^2)_{N_2} = -b_{44}$. At the normal minimum we have $M_A^2 = M_{H^\pm}^2 + b_{44} v^2$, M_A^2 being the squared mass of the pseudoscalar. Then, in this case, eq. (2.18) gives the difference of the values of the potential at the spontaneous CP breaking stationary point and at the normal one, and reduces to

$$V_{CP} - V_N = \frac{M_A^2}{2 v^2} [(v''_1 v_2 - v''_2 v_1)^2 + \delta^2 v_2^2] \quad (2.19)$$

It is clear that if there exists a normal minimum, M_A^2 is positive and the CP stationary point is above the normal minimum. It was also shown in ref.⁹ that the CP stationary point is necessarily a saddle point, analogously to what happens with the CB case.

III. FCNC AND STABILITY

In this section we want to stress a result obtained in⁸: flavor conservation can be achieved by demanding only natural CP conservation and no CB in the absence of fermions, that is, the potentials possessing only CP and CB invariant minima are consistent with the absence of flavor changing neutral currents (FCNC) in the tree level Yukawa couplings. The problem we are addressing is how to force the CB and CP stationarity conditions to have no solution regardless of what the values of the parameters may be. The number of ways to accomplish it is obviously huge. However, the conditions chosen will be useless if they are not preserved by renormalisation. Thus, the only safe way to do it is by means of imposing some kind of symmetry to the potential. The interesting point is that it is possible to enforce the stationary point conditions to have no solution by demanding invariance of the tree-level potential to a Z_2 or to a global $U(1)$ symmetry. That way, the stationary point conditions for the CP and CB cases have no solution at any order in perturbation theory. We have proved in the previous section that a normal minimum is stable in a more general model where CP can be spontaneously broken. However, in that more general model, there is also room for a stable CP minimum. By imposing the Z_2 or a global $U(1)$ symmetry to the potential written in our basis this possibility ceases to exist. The only allowed minimum is the charge and CP conserving one.

The most general Yukawa lagrangian of a general 2HDM gives rise to the appearance of FCNC which are known to be severely constrained by experiment. However, the same symmetries that were used to prevent the existence of the CB and CP stationary points, can now be used to avoid in a natural way the appearance of FCNC at tree level. This can be done by imposing similar symmetries to the appropriate fermion fields.

This way we build two different models which we call A and B¹ in¹¹, where model A is based on a Z_2 symmetry and model B is based on a global $U(1)$ symmetry, softly broken by a dimension two term (otherwise an axion would be produced). They both have 7 independent parameters, are tree-level stable and renormalizable. There are no differences between the two models in the gauge and Yukawa sectors. However, they present quite a different set of Feynman rules in the scalar sector. For instance, the difference between the strength of the vertex hH^+H^- in the two models is

$$(g_{hH^+H^-})_A - (g_{hH^+H^-})_B = 2i \frac{M_A^2}{v^2} \frac{\cos(\alpha + \beta)}{\sin(2\beta)} \quad (3.1)$$

where h is the lightest CP-even scalar.

To finish this section, we would like to point out that whereas in model A the requirement of boundness from below is automatically fulfilled, in model B the same requirement needs a condition between the masses that reads

$$M_h^2 + M_H^2 \geq M_A^2 \quad (3.2)$$

where H is the heaviest CP-even Higgs.

¹In ref.⁸ we have named those models I and II. However, this notation was misleading since the same numbering is used for the different Yukawa lagrangians of 2HDM.

IV. CONCLUSION

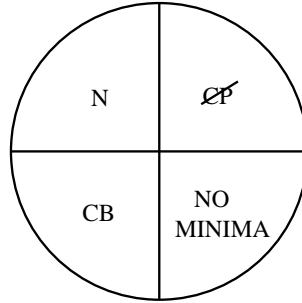


FIG. 1. The quiet world of Two-Higgs doublet models.

In fig.1 we present the result of our work for the potential that does not break CP explicitly. We have shown that the four different worlds do not intersect each other. Each region corresponds to a theory perfectly stable at tree level. Once the world has chosen to be in one of those minima it will remain there unless the values of the parameters of the potential change. If we are for instance in the normal minimum then the model is protected against electric charge or CP spontaneous breaking. In other words, if the model has a minimum preserving $U(1)_{em}$ and CP, that minimum is global. In this way, there is absolutely no possibility of tunneling to deeper minima, and, for instance, the masslessness of the photon is guaranteed in these models. Notice however that the same is true for the CP or the CB minimum. If a CB or a CP minimum exists, it is now the global one and it is perfectly stable. Again, no tunneling occurs.

Charge breaking would be disastrous but there is considerable interest, from cosmologists to particle physicists, in models with the possibility of spontaneous CP violation. We have determined that this cannot happen for those ranges of parameters that lead to Normal minima. However, we have also established a very precise condition for spontaneous CP breaking to occur: CP is spontaneously broken if and only $V'^T B_{CP}^{-1} V' > 0$ ². In these circumstances the 2HDM no longer has a normal minimum, nor a charge breaking one, as we have shown.

In the most general potential there are just the normal minima (CP violating ones) and the CB minimum. There we have proved that the same result holds. It is interesting to point out the following aspect of these results. If one observes equations (2.15) and (2.17), one sees that the difference in the depth of the potential between the normal minimum and the CB stationary point is controlled by the charged Higgs squared mass. On the other hand, equation (2.19) shows that the potential depth difference between the CP and the normal stationary points is proportional to the pseudoscalar squared mass. That is, the depth of the potential at a stationary point that breaks a given symmetry, relative to the normal minimum, depends, in a very straightforward way, on the mass of the scalar particle directly linked with that symmetry.

To finish, let us also stress that our conclusions are absolutely general, independent of particular values of the parameters of the theory, obviously. They hold for any of the more restricted models considered in ref.⁸. It is simple to recover the conditions presented in that reference to avoid CP minima by analyzing the matrix B_{CP} . We remark that the Higgs potential of the Supersymmetric Standard Model (SSM) is also included in the potentials we studied - in fact, it corresponds to the case $b_{11} = b_{22} = -b_{12}/2 = M_Z^2/(2v^2)$, $b_{33} = b_{44} = 2 M_W^2/v^2$ and the remaining b parameters set to zero, following the conventions of ref.¹³. So we could conclude that at tree-level, the Supersymmetric Higgs

²In reference¹² CP violating quantities involving only the Higgs sector were derived in models with explicit CP violation.

potential is safe against charge of CP violation, though this would not preclude charge, colour or CP breaking arising from other scalar fields present in those models. However, we must be cautious: it has been shown¹⁴ that one-loop contributions to the minimization of the potential have an enormous impact on charge breaking bounds in Supersymmetric models. Also, it was recently shown¹⁵ that unless one performs a full one-loop calculation (both for the potential and the vevs, in both the CB potential and the “normal” one) the bounds one obtains can be overestimated. Therefore, we urge caution in applying these conclusions to the SSM. Nevertheless one would expect the one-loop contributions to be much less important in the non-supersymmetric 2HDM due to the much smaller particle content of the latter theory.

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V. REFERENCES

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- ¹ T.D. Lee, *Phys. Rev.* **D8** (1973) 1226; G.C. Branco and M. N. Rebelo, *Phys. Rev.* **D22** (1980) 2901.
² L.McLerron *et al*, *Phys. Lett.* **B256** (1991) 451.
³ M. Sher, *Phys. Rep.* **179** (1989) 273; G.C. Branco, L. Lavoura and J.P. Silva, *CP Violation* (Oxford University Press, Oxford, England, 1999). I. F. Ginzburg and M. Krawczyk, [arXiv:hep-ph/0408011](https://arxiv.org/abs/hep-ph/0408011).
⁴ S. Davidson and H. Haber, [hep-ph/0504050](https://arxiv.org/abs/hep-ph/0504050).
J. F. Gunion and H. Haber [hep-ph/0506227](https://arxiv.org/abs/hep-ph/0506227).
⁵ J.M. Frère, D.R.T. Jones and S. Raby, *Nucl. Phys.* **B222** (1983) 11.
⁶ L. Alvarez-Gaumé, J. Polchinski and M. Wise, *Nucl. Phys.* **B221** (1983) 495; J.P. Derendinger and C.A. Savoy *Nucl. Phys.* **B237** 307; C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quirós, *Nucl. Phys.* **B236** (1984) 438; M. Claudson, L.J. Hall and I. Hinchliffe, *Nucl. Phys.* **B228** (1983) 501; M. Drees, M. Glück and K. Grassie, *Phys. Lett.* **B157** (1985) 164; J.F. Gunion, H.E. Haber and M. Sher, *Nucl. Phys.* **B331** (1988) 320.
⁷ U. Ellwanger and C. Hugonie, [hep-ph/9811386](https://arxiv.org/abs/hep-ph/9811386); S. Abel and T. Falk, *Phys. Lett.* **B444** (1998) 427; S. Abel and C. Savoy, *Phys. Lett.* **B444** (1998) 119; S. Abel and B. Allanach, *Phys. Lett.* **B431** (1998) 339. OPAL Collaboration, *Eur. Phys. Jour* **C7** (1999) 407; *ibid.*, *Eur. Phys. Jour* **C12** (2000) 567.
⁸ J. Velhinho, R. Santos e A. Barroso, *Phys. Lett.* **B322** (1994) 213.
⁹ P.M. Ferreira, R. Santos and A. Barroso, *Phys. Lett.* **B603** (2004) 219.
¹⁰ A. Barroso, P.M. Ferreira and R. Santos, [hep-ph/0507224](https://arxiv.org/abs/hep-ph/0507224).
¹¹ A. Barroso, L. Brucher and R. Santos. *Phys. Rev. D* **60** (1999) 035005; L. Brucher and R. Santos, *Eur. Phys. J. C* **12** (2000) 87.
¹² L. Lavoura and J. P. Silva, *Phys. Rev. D* **50** (1994) 4619 [[arXiv:hep-ph/9404276](https://arxiv.org/abs/hep-ph/9404276)].
¹³ D.J. Castaño, E.J. Piard and P. Ramond, *Phys. Rev.* **D49** (1994) 4882.
¹⁴ G. Gamberini, G. Ridolfi and F. Zwirner, *Nucl. Phys.* **B331** (1990) 331.
¹⁵ P.M. Ferreira, *Phys. Lett.* **B509** (2001) 120; P.M. Ferreira, *Phys. Lett.* **B512** (2001) 379.