African Journal Of Mathematical Physics Vol **3** *No 1 (2006)145-155*

Confining Interquark Potentials Driven by Scalar Field in Non Abelian Gauge Theories: A Review

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Abstract

In this review paper, we revisit in Part I some of the most recent work on confinement in 4d gauge theories with a massive scalar field (dilaton). Emphasis is put on the derivation of confining analytical solutions to the Coulomb problem versus dilaton effective couplings to gauge terms. It is shown that these effective theories can be relevant to model quark confinement and may shed some light on confinement mechanism. In part II, we solve the semi-relativistic wave equation, for Dick interquark potential using the Shifted-l expansion technique (SLET) in the heavy meson sector. The results of this study proves that phenomenological investigation of this mechanism is more than justified and deserves more efforts.

I. INTRODUCTION

Full understanding of the QCD vacuum structure and color confinement mechanism are still lacking. Despite enormous amount of work performed over more than thirty years, particularly in lattice simulations of QCD, direct derivation of confinement from first principles remain still elusive, and there is no totally convincing proposal about its generating mechanism. On the other hand, it is known that the vacuum topological structure of theories with dilaton field is drastically changed compared to the non dilatonic ones[1]. Therefore much about confinement might be learned from such theories, particularly string inspired ones. Indeed the appearance of fundamental scalars with direct coupling to gauge curvature terms in string theories offers a challenge with attractive implications in four-dimensional gauge theories [1]. Besides, since color confinement can be signaled through the behavior of the interaction potential at large distances, it was suggested in[3] that an effective coupling of a massive dilaton to the 4-dimensional gauge fields may provide an interesting mechanism which accommodates both the Coulomb and confining phases. The derivation performed in[3,4] suggest a new scenario to generate color confinement. This scenario may be considered as a challenge to the mechanism of monopole condensation.

[1]The dilaton is an hypothetical scalar particle predicted by string theory and Kaluza-Klein type theories. In string theory, its expectation value probes the strength of the gauge coupling[2].

The outline of this review paper is as follows. In part I, we describe the influence of the dilaton on a low energy gauge theory and look into the problem how dilatonic degrees of freedom modifie Coulomb potential and how a confining phase shows up. We summarize the results of several recent works by presenting the corresponding analytic solutions of the field equations and and comment their confinement features. In Part II, it seems to us more than justified to dedicate some efforts to phenomenological investigations. We present the SLET technique used to study of Dick interaction potential in the heavy quarkonium spectra. Then, we devote a section to the analysis and discussion of the obtained results. Finally, we draw our concluding remarks.

II. PART I: THE LOW ENERGY EFFECTIVE THEORY

The impact of dilaton on a 4d effective nonabelian gauge theory is described by a Lagrangian density:

$$\mathcal{L}(\phi, A) = -\frac{1}{4F(\phi)} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - J^\mu_a A^a_\mu \tag{2.1}$$

where ϕ is the dilaton field and $G^{\mu\nu}$ is the standard field strength tensor of the theory. $V(\phi)$ denotes the dilaton potential and $F(\phi)$ represents the coupling function depending on the dilaton field. Several forms of $F(\phi)$ have been proposed in literature. The most popular one $F(\phi) = e^{-k\frac{\phi}{f}}$ occurred in string theory and Kaluza-Klein theories[2].

The problem of the Coulomb gauge theory augmented with dilatonic degrees of freedom in (1) is analyzed as follows: we first consider a point like static Coulomb source which is defined in the rest frame by the current:

$$J^\mu_a = g\delta(r) C_a \nu^\mu_0 = \rho_a \eta^\mu_0 \tag{2.2}$$

where C_a is the expectation value of $SU(N_c)$ generator. The field equations emerging from the static configuration (2) are given by:

$$\left[D_\mu, F^{-1}(\phi) G^{\mu\nu}\right] = J^\nu \tag{2.3}$$

and

$$\partial_\mu \partial^\mu \phi = -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{4}\frac{\partial F^{-1}(\phi)}{\partial \phi} G^{\mu\nu}_a G^{\nu\mu}_a \tag{2.4}$$

At this stage, by setting $G^{0i}_a = E^i \chi_a = -\nabla^i \Phi_a$, after some algebra, we derive the chromo-electric field:

$$E_a = \frac{Q^a_{eff}(r)}{r^2} \tag{2.5}$$

where the effective charge is defined by $Q^a_{eff}(r) = \left(g\frac{C_a}{4\pi}\right) F(\phi(r))$.

From Eq.(5), we learn that it is the running of the effective charge that makes the potential stronger than the Coulomb potential. In other words, Coulomb spectrum is recovered if the effective charge did not run. Thereby the interquark potential reads as[4],

$$U(r) = 2\widetilde{\alpha}_s \int \frac{F(\phi(r))}{r^2} dr \tag{2.6}$$

with $\alpha_s = \frac{g^2}{4\,\pi}$ and $\widetilde{\alpha} = \frac{\alpha_s}{8\pi}\left(\frac{N_c - 1}{2N_c}\right)$

The formula (6) remarkable since it provides a direct relation between the interquark potential and the coupling function $F(\phi(r))$. Moreover, it shows that existence of a confining phase in this effective theory is subject to the condition,

$$\lim_{r\to\infty} rF^{-1}(\phi(r)) = finite \tag{2.7}$$

Note that our main objective is to solve the field equations of motion (3) and (4) and determine analytically $\phi(r)$ and $\Phi_a(r)$. For this, $F(\phi)$ and $V(\phi)$ have to be fixed. In the sequel the dilaton potential is set to $V(\phi)=\frac{1}{2}m^2\phi$. Below, we will briefly describe the main features of some recent models and present their analytical solutions.

A. Dick Model

In this effective theory, Dick used the form: $\frac{1}{F(\phi)}=\frac{\phi^2}{f^2+\beta\,\phi^2}$ with f a coupling scale characterizing the strength of the scalar-gluon coupling. and β is a parameter is the range $0<\beta<1$. He derived the radial dependence of the dilaton field and the interquark potential (up to a color factor)[3]:

$$\phi(r)=\pm\frac{1}{r}\sqrt{\frac{k}{m}+(y_0^2-\frac{k}{m})exp(-2mr)}$$

$$V(r)=[\frac{\beta\,g^2}{4\pi\,r}-gf\sqrt{\frac{N_c}{2\,(N_c-1)}}ln[e^{2mr}-1+\frac{m}{k}y_o^2]$$

With the abbreviation: $k^2=\frac{\alpha_s\,f^2}{8\,\pi}(\frac{N_c-1}{N_c})$

It is remarkable that the potential $V(r)$ comes with the required behavior: a first term which accommodates the Coulomb interaction at short distances, and a second term linearly rising in the asymptotic regime with a string tension [2] $\sigma\sim g\,m\,f$ which depends on the dilatonic degrees of freedom m and f.

B. Cornwall-Soni Model

Cornwall-Soni observed that the coupling term $\phi G^a_{\mu\nu}G^{\mu\nu}_a$ can be used to account for a glueball decay. They were the first to motivate such term as a low energy contribution to effective models for QCD. In this case, $\frac{1}{F(\phi)}=\frac{\phi}{f}$[6].

Analytical Solutions were found for $r\rightarrow\infty$[7], $\phi(r)=\left[\frac{\alpha_s\,f\,(N_c-1)}{16\,\pi\,m^2\,N_c}\right]^{\frac{1}{3}}r^{-\frac{4}{3}}$, and $V(r)=-3\,g\frac{N_c-1}{2\,N_c}\left[\frac{g\,f^2\,N_c\,m^2}{\pi\,(N_c-1)}\right]^{\frac{1}{3}}r^{\frac{1}{3}}$

These formulas show that, at large distances, confinement is probed through an interquark potential proportional to $V(r)\sim r^{1/3}$. This confining term could be considered as nonperturbative correction to the Coulomb phase.

C. Chabab-Sanhaji Model

In this work, we proposed a low energy effective field theory from which some popular phenomenological potentials are derived. To this end, we used the following coupling function $F(\phi)=\left(1-\beta\frac{\phi^2}{f^2}\right)^{-n}$[8]. By substituting $F(\phi)$ in Eq. (3,4), the field equations were found too complicated to integrate analytically. However, as in Cornwall-Soni Model, since the focus is on the long range behavior of the dilaton field

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[2] In the massless case, $V(\phi)=0$, solutions of the field equations reduced to: $\phi(r)=\pm\left(\frac{g\,f}{2\pi}\right)\sqrt{\frac{N_c-1}{N_c}}\,r^{\frac{-1}{2}}$, $V(r)=\frac{g^2\,\beta\,(N_c-1)}{8\,\pi\,r N_c}-\frac{f\,g}{2}\sqrt{\frac{N_c-1}{N_c}}\,r$

and on how it modifies the Coulomb phase, the analysis is restricted to the infrared region. Thus, the asymptotic solutions are found to be,

$$\phi = \left[\frac{f^2}{\beta} - \left(\frac{\beta}{f^2}\right)^{\frac{-n}{n+1}} \left(\frac{2n\alpha_s}{m^2}\right)^{\frac{1}{n+1}} \left(\frac{1}{r}\right)^{\frac{4}{n+1}}\right]$$

and the chromo-electric potential:

$$\Phi_a(r) = -\frac{gC_a}{4\pi}\left(\frac{2n\alpha_s}{m^2 f^2}\right)^{\frac{-4n}{n+1}} \frac{n+1}{3n-1} r^{\left(\frac{3n-1}{n+1}\right)}$$

We see that the occurrence of confinement depends on the parameter n and imply that our effective theory can serve to model quark quark confinement when $n \in \left[\frac{1}{3}, 1\right]$.

On the other hand, we also notice that by selecting specific values of n, the following known interquark potentials are reproduced:

- $n = 1 \Rightarrow$ linear term of Cornwall potential.
- $n = 11/29 \Rightarrow$ Martin's potential[13].
- $n = 3/5 \Rightarrow$ Song-Lin, or Motyka-Zalewski' potential[32].
- $n = 5/9 \Rightarrow$ Turin potential[15].

Therefore these potential models, which gained credibility only through their confrontation to the hadron spectrum, are now supplied with a theoretical framework since they can be derived from a low energy effective theory.

D. Generalized Dick Model

The authors of ref.[37] used the coupling function proposed by Dick to power of α: $\frac{1}{F(\phi)} = (\frac{\phi}{\Lambda})^\alpha$
In the case of of a massless dilaton, they found the following electric solutions, $\phi(r) = \Lambda\, A_\alpha\, (\frac{1}{\Lambda r})^{\frac{2}{2+\alpha}}$, and $V(r) = \Lambda\, B_\alpha (\Lambda\, r)^{\frac{\alpha-2}{2+\alpha}}$where $A_\alpha = [\frac{g}{4\pi}(\frac{\alpha}{2}+1)]^{\frac{2}{2+\alpha}} (\frac{N_c-1}{2\,N_c})^{\frac{1}{2+\alpha}}$, and $B_\alpha = \frac{g^2}{4\pi}\,\frac{N_c-1}{2\,N_c}\,\frac{1}{A_\alpha^\alpha}\,\frac{\alpha+2}{2-\alpha}$
In this model, confinement shows us for values of $\alpha > 2$. In particular, a potential with linear behaviour is realized when $\alpha \to \infty$.

In the case wher $mr \to \infty$, they show that solutions are reduced to, $\phi(r) = C_\alpha\, r^{-\frac{4}{\alpha+2}}$ and $V(r) = D_\alpha\, r^{\frac{3\alpha-2}{\alpha+2}}$, with
$C_\alpha = [\frac{\alpha\, g^2\, \Lambda^\alpha}{64\, m^2\, \pi^2}\,\frac{N_c-1}{N_c}]^{\frac{1}{\alpha+2}}$, $D_\alpha = -\frac{g^2}{4\,\pi}\,\frac{N_c-1}{2N_c}(\frac{\Lambda}{C_\alpha})^\alpha\,\frac{\alpha+2}{3\alpha-2}$.
In this case, $V(r)_{r\to\infty} \to \infty$ for $\alpha > \frac{2}{3}$.

III. PART II: PHENOMENOLOGICAL INVESTIGATION

The problem of calculating the spin-averaged energy levels heavy-mesons is a very old subject, but still an important theme in the existing literature. At present much experimental material on the masses of the ground and excited states of heavy quarkonia has been accumulated. By confronting theoretical predictions to experimental data, one can extract important information on the form of the potential of quark-quark interaction and probes the strength and Lorentz structure of its confining part. This can improve our knowledge of the main features of quantum chromodynamics (QCD) dynamics: asymptotic freedom and quark confinement. At present, much attention has been focused on QCD motivated quark models, specially those describing successfully meson spectroscopy. At short distances, QCD suggests a Coulomb-type potential $-\frac{4}{3}\frac{\alpha_s}{r}$, while at large distances a confining phase is expected. α_s is the quark-gluon coupling constant and r is the interquark distance. This behaviour is confirmed by Lattice calculations[27] and string models[28]. However, only Lattice gauge Wilson loop computations convincingly identified the interquark potential in the range of distances $0.1\ fm < r < 2\ fm$.[3] But, serious attempts have still to be made with the aim of highlighting which form of quark potential model is more reliable and more related to QCD.[5]. Now it is firmly established that the combination of Coulomb-type potential, based on the one gluon exchange plus a confining potential with a long range linear rising provides a good fit to hadron spectra[29]. [4] However, one would be hard pressed to say that hadron spectra are well understood. Only the ground state S, and P wave multiplets are filled and too many of the $Q\bar{Q}$ states predicted by quark potential models have yet to be seen. Therefore, until color vacuum structure and quark confinement mechanism are better understood, more states have been sited, their properties measured and the strength and Lorentz structure of the interquark potential definitely settled from QCD first principles, we cannot say that the subject is completed and that our knowledge of hadronic world is perfect.

Therefore, our aim in this part of this review paper is to dedicate more efforts to understand this confinement scenario through the phenomenological investigation of Dick potential $V_D(r)$ in the heavy quarkonium sector. This problem will be addressed as in the work[11]. Therein, it has been demonstrated that the shifted-l expansion (SLET), where l is the angular momentum, provides a powerful analytic technique for determining the bound states of the semi-relativistic wave equation consisting of two quarks of masses m_1, m_2 and total binding meson energy M in any spherically symmetric potential. It consists of using $1/\bar{l}$ as a pseudo-perturbation parameter, where $\bar{l} = l - \beta$ and β is a suitable shift. This method yields very accurate and rapidly converging energy eigenvalue series. It also handles highly excited states which pose problems for variational methods[12]. Moreover, relativistic corrections are included in a consistent way.

A. SLET for the semi-relativistic wave equation with a spherically symmetric potential

There are different methods to calculate the bound state energies of the semi-relativistic wave equation:

$$[(p^2+m_1^2\,)^{1/2}+(p^2+m_2^2\,)^{1/2}+V(r)-M\,]\,\psi(r)=0\,. \tag{3.1}$$

where M is the total binding meson mass. An expansion in the powers of $(v/c)^2$ up to two terms in the semi-relativistic equation which is a combination of relativistic kinematics with some static interaction potential yields[11]:

$$\left\{\frac{p^2}{2\mu}\ -\ \frac{p^4}{8\nu}\ +\ V(r)\ \right\}\psi_{n\ell}(r)\ =\ E_{n\ell}\ \psi_{n\ell}(r)\ , \tag{3.2}$$

where $E_{n\ell} = M_{n\ell} - m_1 - m_2$ stands for the Salpeter binding energy for two bound interacting quarks of masses m_1 and m_2, $\mu = m_1 m_2/(m_1 + m_2)$ represents the reduced mass and $\nu = m_1^3 m_2^3/(m_1^3 + m_2^3)$ is a useful parameter. In this reduction formalism the spherically symmetric potential $V(r)$ which represents the interaction between the two particles remains unspecified. In the literature, the second term in Eq.(9) is treated as a perturbation by using trial wave functions. In this work, in order to obtain a Schrödinger-like equation, this term is handled via the reduced Schrödinger equation:

$$p^4 = 4\mu^2\Big[M - m_1 - m_2 - V(r)\Big]^2. \tag{3.3}$$

Thus, the second order Schrödinger−like equation to order $(v/c)^2$ becomes

$$\left\{\frac{p^2}{2\mu}-\frac{1}{2\eta}[E_{n\ell}^2+V^2(r)-2E_{n\ell}V(r)]+V(r)\right\}\psi_{n\ell}(r)=E_{n\ell}\psi_{n\ell}(r), \tag{3.4}$$

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[3]These computations are performed without light dynamical quarks.

[4]We refer the reader to the report of Lucha et al.[29] which contains many references on the subject.

where $\eta = \nu/\mu^2$. Writing the operator p^2 in the spherical polar coordinates

$$p^2 = -\frac{d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + \frac{l(l+1)}{r^2}, \tag{3.5}$$

and for states of definite orbital angular momentum l, define the reduced radial wave function $R_{n\ell}(r)$ by $\psi_{n\ell}(r) = r^{-1}R_{n\ell}(r)Y_{lm}(\theta,\phi)$, then after some algebra Eq.(11) can be transformed to a Schrödinger-like equation (in units $\hbar = c = 1$) for the radial wave function $R_{n\ell}(r)$:

$$\left[-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + \gamma(r) + \frac{E_{n\ell}V(r)}{\eta}\right] R_{n\ell}(r) = (\frac{E_{n\ell}^2}{2\eta} + E_{n\ell})R_{n\ell}(r), \tag{3.6}$$

with $\gamma(r) = V(r) - V^2(r)/2\eta$.

If the angular momentum l is shifted through the relation $l = \bar{l} + \beta$, Eq.(13) becomes,

$$\left[-\frac{1}{2\mu}\frac{d^2}{dr^2} + \bar{l}^2[\frac{1+\bar{l}(2\beta+1)/\bar{l} + \beta(\beta+1)/\bar{l}^2}{2\mu r^2} + \frac{\gamma(r)}{Q} + \frac{E_{n\ell}V(r)}{Q\eta}]\right] R_{n\ell}(r) = (\frac{E_{n\ell}^2}{2\eta} + E_{n\ell})R_{n\ell}(r), \tag{3.7}$$

where n in this paper is the radial quantum number, and Q is a constant that scales the potential $V(r)$ at large-l limit and is set for any specific choice of l and n, equal to $\bar{l}^2$ at the end of the calculations.

The systematic procedure of SLET begins with shifting the origin of the coordinate through the definition:

$$x = \bar{l}^{1/2}(r - r_o)/r_o \tag{3.8}$$

where r_o is currently an arbitrary point to perform Taylor expansion about, with its particular value to be determined below. Expansion about this point yield:

$$\frac{1}{r^2} = \sum_{j=0}^{\infty}(-1)^j\frac{(j+1)}{r_o^2}x^j\bar{l}^{-j/2}, \tag{3.9}$$

$$V(x(r)) = \sum_{j=0}^{\infty}\frac{d^jV(r_o)}{dr_o^j}\frac{(r_ox)^j}{j!}\bar{l}^{-j/2}, \tag{3.10}$$

$$\gamma(x(r)) = \sum_{j=0}^{\infty}\frac{d^n\gamma(r_o)}{dr_o^j}\frac{(r_ox)^j}{j!}\bar{l}^{-j/2}. \tag{3.11}$$

It should be mentioned here that the re-scaled coordinate in Eq.(15) has no effect on the energy eigenvalues, which are coordinate-independent. It just facilitates the calculations of both the energy eigenvalues and wave functions. It is also convenient to expand $E_{n\ell}$ as, $E_{n\ell} = \sum_{i=0}^{\infty} E_i\bar{l}^{-i}$. By substituting Eqs.(16-18) into Eq.(14) and expanding around $x = 0$ in powers of x and $\bar{l}$, one gets:

$$\begin{aligned}&\left[\frac{-1}{2\mu}\frac{d^2}{dx^2} + \frac{1}{2\mu}(\bar{l} + (2\beta+1) + \frac{\beta(\beta+1)}{\bar{l}})(1 - \frac{2x}{\bar{l}^{1/2}} + \frac{3x^2}{\bar{l}} - \cdots)\right.\\ &+\frac{r_o^2\bar{l}}{Q}(\gamma(r_o) + \frac{\gamma'(r_o)r_ox}{\bar{l}^{1/2}} + \frac{\gamma''(r_o)r_o^2x^2}{2\bar{l}} + \frac{\gamma'''(r_o)r_o^3x^3}{6\bar{l}^{3/2}} + \cdots)\\ &\left.+\frac{r_o^2\bar{l}}{\eta Q}(V(r_o) + \frac{V'(r_o)r_ox}{\bar{l}^{1/2}} + \cdots)(E_o + \frac{E_1}{\bar{l}} + \frac{E_2}{\bar{l}^2} + \cdots)\right]\phi_{n\ell}(x)\\ &= \mathcal{E}_{n\ell}\phi_{n\ell}(x),\end{aligned} \tag{3.12}$$

where the prime of $V(r_o)$ and $\gamma(r_o)$ denotes derivatives with respect to r_o,

$$\mathcal{E}_{n\ell} = \frac{r_o^2}{Q}\left[\bar{l}(E_o + \frac{E_o^2}{2\eta}) + (E_1 + \frac{E_oE_1}{\eta}) + (E_2 + \frac{E_oE_2}{\eta} + \frac{E_1^2}{2\eta})\frac{1}{\bar{l}} + (E_3 + \frac{E_oE_3}{\eta} + \frac{E_1E_2}{\eta})\frac{1}{\bar{l}^2} + \cdots\right]. \tag{3.13}$$

Equation (19) resembles exactly to Schrödinger-like equation for the one-dimensional anharmonic oscillator and has been investigated in detail for spherically symmetric potentials in[30]:

$$\left[-\frac{1}{2\mu}\frac{d^2}{dx^2} + \frac{1}{2}\mu\omega^2x^2 + \epsilon_o + P(x)\right]\chi_{n\ell}(x) = \lambda_{n\ell}\chi_{n\ell}(x). \tag{3.14}$$

To estimate $P(x)$ and ϵ_o, we simply match the terms of Eqs. (19,20) with those in Eq.(21). Consequently, the final analytic expression for the energy eigenvalues appropriate to a semi-relativistic particle is given by,

$$\begin{aligned}\mathcal{E}_{n\ell} = \bar{l}&\left[\frac{1}{2\mu} + \frac{r_o^2V(r_o)E_o}{\eta Q} + \frac{r_o^2\gamma(r_o)}{Q}\right] \\ &+\left[\frac{(2\beta+1)}{2\mu} + \frac{r_o^2V(r_o)E_1}{\eta Q} + (n+\frac{1}{2})w\right] \\ &+\frac{1}{\bar{l}}\left[\frac{\beta(\beta+1)}{2\mu} + \frac{r_o^2V(r_o)E_2}{\eta Q} + \alpha_1\right] \\ &+\frac{1}{\bar{l}^2}\left[\frac{r_o^2V(r_o)E_3}{\eta Q} + \alpha_2\right],\end{aligned} \tag{3.15}$$

where α_1 and α_2, appearing as corrections to the leading order of the energy expression, are defined and listed in the appendix of ref.[31], while the relevant quantities $\epsilon^{'}$s and $\delta^{'}$s corresponding to the semi-relativistic equation are given in[11]. A quantitative estimate for the energy terms in Eq.(22) can be derived by comparing terms of the same order in $\bar{l}$ in Eq.(20). A straightforward calculations show the following results,

$$E_o = V(r_0) - \eta + \sqrt{\eta^2 + \frac{\eta Q}{\mu r_0^2}}, \tag{3.16}$$

$$E_2 = \frac{Q\alpha_{(1)}}{r_0^2\left(1 + \frac{E_0 - V(r_0)}{\eta}\right)}, \tag{3.17}$$

$$E_3 = \frac{Q\alpha_{(2)}}{r_0^2\left(1 + \frac{E_0 - V(r_0)}{\eta}\right)}. \tag{3.18}$$

Here, β is chosen so that the next contribution to the leading term in the energy eigenvalue series vanishes, i.e., $E_1 = 0$, which implies that, $\beta = -1/2 - \mu(n + 1/2)\omega$, with $\omega = \frac{1}{\mu}\left[3 + r_0V''(r_0)/V'(r_0) - \mu r_0^4V'(r_0)^2/(Q\eta)\right]^{1/2}$ and where Q satisfies the equation, $Q = \frac{\mu}{2\eta}\left[r_0^2V'(r_0)\right]^2(1+\xi)$, with $\xi = \sqrt{1 + [2\eta/r_0V'(r_0)]^2}$ On the other hand, r_0 is determined by minimizing E_0,

$$\frac{dE_0}{dr_0} = 0 \quad \text{and} \quad \frac{d^2E_0}{dr_0^2} > 0, \tag{3.19}$$

Collecting these terms and carrying out the mathematics, one gets

$$1 + 2\ell + \mu(2n+1)\omega = r_0^2V'(r_0)\left(\frac{2\mu}{\eta} + \frac{2\xi\mu}{\eta}\right)^{1/2} \tag{3.20}$$

which is an explicit equation in r_0. Finally, by substituting the expressions of Eqs.(23)-(25) into Eq.(20) we immediately derive the following formula for the energy eigenvalues,

$$E_{n\ell} = E_0 + \frac{\alpha_{(1)}}{r_0^2\left(1 + \frac{E_0 - V(r_0)}{\eta}\right)} + \frac{\alpha_{(2)}}{r_0^2\left(1 + \frac{E_0 - V(r_0)}{\eta}\right)\bar{l}} + O\left[\frac{1}{\bar{l}^2}\right]. \tag{3.21}$$

from which we deduce the bound state mass: $M_{n\ell} = E_{n\ell} + m_1 + m_2$.

B. Analysis, Results and Discussion

We consider Dick interquark potential $V_D(r)$ given by,

$$V_D(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \frac{4}{3}gf\sqrt{\frac{N_c}{2(N_c-1)}}\ln[exp(2mr)-1] \tag{3.22}$$

Examination of the previous section shows that there are five input parameters to be specified: m_c, m_b, m, f and α_s. In our numerical analysis, we set the charm and bottom quark masses to the values $m_c = 1.89$ GeV and $m_b = 5.19$ GeV. For the QCD coupling constant, in contrast to the Lattice potentials which use the same effective coupling in the description of heavy quarkonia, most potential models take into account the running of α_s[18]. We fix the renormalization scale to $\lambda = 2\mu$ where μ is the reduced mass,

$$\mu = \frac{m_1 m_2}{m_1 + m_2}, \tag{3.23}$$

Then, from the leading order formula,

$$\alpha_s(\lambda) = \frac{\alpha_s(m_z)}{1 - (11 - \frac{2}{3}n_f)[\alpha_s(m_z)/2\pi]ln(m_z/\lambda)}, \tag{3.24}$$

combined to the world experimental value $\alpha_s(m_z) = 0.118$ we obtain,

$$\alpha_s = 0.31, \qquad \alpha_s = 0.20 \tag{3.25}$$

for the charmonium and bottomonium systems respectively, while $\alpha_s = 0.22$ for the $b\bar{c}$ quarkonia. On the other hand, the dilaton parameters m and f are considered as free in our analysis and are obtained by fitting experimental data of $c\bar{c}$ and $b\bar{b}$ spectrum. An excellent fit emerge when the following values are assigned

$$m = 57\ MeV \qquad gf\sqrt{\frac{N_c}{2(N_c-1)}} = 430\ MeV.^5 \tag{3.26}$$

The results of our analysis for the spin-averaged energy levels of interest are shown in Tables (1,2). In all cases, where comparison with experiment is possible, we found generally good agreement . Next step, to check the consistency of our predictions, we estimate the bound states energies of the $b\bar{c}$ quarkonia. These states are expected to be produced at LHC and Tevatron. Their observation should provide an excellent test to discriminate between various theoretical techniques used to probe nonperturbative hadrons properties. In table 3 we list our calculated spectrum. Our estimate for the B_c mass, the lowest pseudoscalar S-state of the spectra, is compatible with the experimental value reported by CDF collaboration[17] and with the result of the recent Lattice calculations[33,34]. As to the higher states masses, they compare favorably with other predictions based on QCD sum-rules[18,19] or potential models[25] -[35]. In conclusion, Dick interquark potential (29) is tested successfully to fit the spin-averaged $c\bar{c}$, $b\bar{b}$ and $b\bar{c}$ spectrum. In view of these results, it is quite encouraging to pursue application of $V_D(r)$ and other interquark potentials emerging from such a low energy effective gauge theory. It is also appealing to investigating spin-dependant corrections responsible for fine and hyperfine splitting of the energy levels and decays. On the other hand, as a by-product, this analysis allows to probe physics beyond the standard model in relation to hadron spectroscopy. Indeed, our estimate for the mass of the dilaton resulting from a fit to the existing experimental data of heavy quarkonia, lies in the range of masses proposed in[39,40]. Since a unique scenario for the dilaton mass is still lacking[38], this determination may shed some light in the search of the dilaton, the discovery of which would be a major step towards the experimental validation of string theory. Indeed, the possibility to identify this hypothetical particle to a fundamental scalar invisible to present day experiments should not be exclude[41,42].

IV. GENERAL CONCLUSION

In summary, We reviewed some of the most recent work on confinement in 4d non abelian gauge theories with a massive scalar field (dilaton) and effective coupling functions to gauge fields. Analytical solutions have been found with confinement feature in the asymptotic regime. Thus, These low energy effective theories can serve well to model quark confinement. Moreover, by using Dick interquark potential in the heavy quarkonium sector, we showed that phenomenological investigation of the confinement generating mechanism suggested by these models is more than justified. Indeed, the obtained results for charmoniun and bottomonium fit well experimental data when the dilaton mass is given a value about 57 MeV. Also, for B_c system, we found that the S-state energy level is close to the value reported by CDF collaboration, while those of excited states agree favorably with predictions of other theoretical works. On the other hand, This analysis allows a test to the physics beyond the standard model in relation to hadron spectroscopy. Indeed, as a by-product, the estimate of the dilaton mass lies in the range of values proposed in[39,40]. This determination may shed some light on the search of the dilaton since, as suggested in[41,42], the possibility to identify this hypothetical particle to a fundamental scalar invisible to present day experiments should not be excluded.

Acknowledgements This work is supported by Morocco research program PROSTARS III, under contract D16/04.

TABLE I. Calculated mass spectra $M_{n\ell}$ of $c\bar{c}$ boundstates (in GeV) from Dick potential[26]

State,$n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State,$n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	3.073	3.068	1P	3.546	3.525
2S	3.662	3.663	2P	3.871	-
3S	4.027	4.028	1D	3.787	3.788

TABLE II. Calculated mass spectra $M_{n\ell}$ of $b\bar{b}$ boundstates (in GeV) from Dick potential[26]

State,$n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State,$n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	9.450	9.446	1P	9.903	9.900
2S	10.014	10.013	2P	10.227	10.260
3S	10.299	10.348	1D	10.129	-

TABLE III. Calculated mass spectra $M_{n\ell}$ of $b\bar{c}$ boundstates (in GeV) from Dick potential[26]

State,$n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State,$n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	6.322	6.40 $\pm$0.39$\pm$0.13	1P	6.767	-
2S	6.876	-	2P	7.072	-
3S	7.181	-	1D	6.994	-

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