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Mott Scattering of Polarized Electrons in a Strong Laser Field

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Abstract

We present analytical and numerical results of the relativistic calculation of the transition matrix element S_{fi} and differential cross sections for Mott scattering of initially polarized Dirac particles (electrons) in the presence of strong laser field with linear polarization. We use exact Dirac-Volkov wave functions to describe the dressed electrons and the collision process is treated in the first Born approximation. The influence of the laser field on the degree of polarization of the scattered electron is reported.

I. INTRODUCTION

Spin is an essential and fascinating complication in the physics of quantum collision theory as well as in many fields of physics. The spin of a particle is a quantum mechanical attribute. Therefore, questions about the spin dependence of atomic reactions tend to probe the underlying theoretical structure very deeply. On the particle side, the technology of spin measurement has improved dramatically over the past years[1],[2]. Improvement in polarized sources allow to produce successfully polarized gas whereas polarized electrons and positrons in (e^+e^-) colliders are commonplace. This area of study has served as a crucial testing grounds in elementary particle physics and in atomic scattering theories and experiments. Methods that have passed this test successfully are now being widely used for a series of practical applications as well. These include the production of atomic data for the modeling of fusion plasma, as well as data needed for astrophysics and laser physics. Whenever the spin direction plays a role, one has to average over all spin orientations in order to describe the experiments properly. Only in recent years has it been found possible to produce electron beams in which the spin has a preferential orientation. They are called polarized electrons beams[4] in analogy to polarized light in which it is the field vectors that have a preferred orientation. There are many reasons for the interest in polarized electrons. One important reason is that in physical experiments one endeavors to define as exactly as possible the initial and/or final states of the systems being considered. Moreover experiments on laser-induced process in strong laser fields well beyond the atomic field strength intensity of about 3.5 10^{16} W/cm^2 have clearly given evidence of relativistic effects[5]. However, the search for spin-specific effects has been rare[6],[7].

The aim of this contribution is to add some new physical insights and to show that the modification of the polarization degree due to the presence of a strong laser field can provide (or not) a clear signature of spin effects in electron-laser interaction. Before we present the results of our investigation concerning laser-assisted Mott scattering of polarized electrons, we first begin by sketching the principal steps of our treatment. For pedagogical purposes, we begin by the most basic results of Mott scattering of polarized electrons in the absence of the laser field using atomic units $(a.u.)$. In atomic units, one has $(\hbar = m_e =$

$e = 1$) where m_e is the electron rest mass. Throughout this work, we shall use atomic units and work with the metric tensor $g^{\mu\nu} = diag(1, -1, -1, -1)$. The scalar Lorentz product of two four-vectors a and b is defined by $a.b = a^\mu b_\mu = a^0 b_0 - \mathbf{a}.\mathbf{b}$ where the quantity in bold face $\mathbf{a}.\mathbf{b}$ stands for the ordinary scalar product in three dimensions. Then, in the presence of a laser field, we compare the results obtained with those obtained in the absence of the laser field. The organization of this paper is as follows. In section 2, we discuss the laser-assisted Mott scattering of polarized electrons in the presence of a laser field. In section 3, we discuss the results we have obtained and we end by a brief conclusion in section 4. We hope to offer a simple pedagogical treatment of spin in relativistic atomic collisions that strips it of its unnecessary mystery. Our approach based upon the helicity formalism leads to a unified treatment that can be applied to other relativistic atomic scattering processes in the absence or in the presence of a laser field. Finally, we would like to emphasize the following : the spin polarization state of a Dirac particle has not to be confused with the polarization of the laser field (which can be linear, circular or elliptical) used to describe the process let aside with the number n of photons exchanged between the Dirac particles and the laser field.

II. MOTT SCATTERING OF POLARIZED ELECTRONS IN THE PRESENCE OF A STRONG LASER FIELD WITH LINEAR POLARIZATION.

Let us consider the simple case where the linearly polarized laser field is described by the four potential

$$A(x) = a_1 \cos(k.x) \tag{2.1}$$

with $k.x = k_\mu x^\mu = \omega t - \mathbf{k}.\mathbf{x}$ where ω and $\mathbf{k}$ are the frequency and the wave vector of the laser field respectively. The electron is described by the Dirac -Volkov wave function[11] solution of the Dirac equation in the presence of a laser field. where the averaged squared potential is such $\overline{A}^2 = \overline{a_1^2} = -\mathbf{a^2/2}$. We turn now to the calculation of the transition amplitude. The interaction of the dressed electron with the central Coulomb field

$$A^\mu_{Coul} = \left(-\frac{Z}{|\mathbf{x}|}, 0, 0, 0\right) \tag{2.2}$$

is considered as a first order perturbation. This is well justified if $Z\alpha \ll 1$ where Z is the atomic number and α is the fine structure constant. Then, the transition matrix element for the transition $(i \to f)$ is

$$S_{fi} = iZ \int_{-\infty}^{+\infty} dt \langle \psi_{q_f}(x) | \frac{\gamma^0}{|\mathbf{x}|} | \psi_{q_i}(x) \rangle \tag{2.3}$$

Let us consider the term

$$\begin{aligned}\overline{\psi}_{q_f}(x)\gamma^0\psi_{q_i}(x) = {}& \frac{\overline{u}(p_f, s_f)}{\sqrt{2Q_f V}} \left[1 + c(p_f) \not{a}_1 \not{k} \cos(\phi)\right] \gamma^0 \left[1 + c(p_i) \not{k} \not{a}_1 \cos(\phi)\right] \frac{u(p_i, s_i)}{\sqrt{2Q_i V}} \\ & \times \exp(-i(q_i - q_f).x - iz\sin(\phi))\end{aligned} \tag{2.4}$$

with

$$c(p) = \frac{1}{2c(k.p)} \quad ; \quad z = \frac{1}{c}\left(\frac{a_1.p_i}{k.p_i} - \frac{a_1.p_f}{k.p_f}\right) \tag{2.5}$$

Transforming the terms containing products of Dirac gamma matrices and invoking the well known result for ordinary Bessel functions

$$e^{[-iz\sin(\phi)]} = \sum_{n=-\infty}^{+\infty} J_n(z) e^{(-in\phi)}, \tag{2.6}$$

one ends up with

$$S_{fi} = \frac{iZ4\pi}{\sqrt{2Q_iV}\sqrt{2Q_fV}} \sum_{n=-\infty}^{+\infty} \frac{\overline{u}(p_f,s_f)\Gamma_n u(p_i,s_i)}{|\mathbf{q_i}+\mathbf{nk}-\mathbf{q_f}|^2} 2\pi\delta(Q_i - Q_f + n\omega) \tag{2.7}$$

where

$$\Gamma_n = \gamma^0 B_{0n} + \gamma^0 \not{k}\not{a}_1 B_{1n} + \not{a}_1\not{k}\gamma^0 B_{2n} - 2k^2a^2\not{k}B_{3n} \tag{2.8}$$

The coefficients B_{0n}, B_{1n}, B_{2n} and B_{3n} are given in[10]. Using the standard procedures of QED, one gets for the unpolarized DCS

$$\frac{d\overline{\sigma}}{d\Omega_f} = \sum_{n=-\infty}^{+\infty} \frac{Z^2}{c^4}\frac{|\mathbf{q_f}|}{|\mathbf{q_i}|}\frac{1}{|\mathbf{q_i}+\mathbf{nk}-\mathbf{q_f}|^4}\frac{1}{2}\mathrm{Tr}\left\{\Gamma_n(c\not{p}_i+c^2)\overline{\Gamma}_n(c\not{p}_f+c^2)\right\}\Big|_{Q_f=Q_i+n\omega} \tag{2.9}$$

whereas the corresponding polarized DCS is given by

$$\frac{d\sigma(\lambda_i,\lambda_f)}{d\Omega_f} = \sum_{n=-\infty}^{+\infty} \frac{Z^2\,|\mathbf{q_f}|}{c^4\,|\mathbf{q_i}|\,|\mathbf{q_i}+\mathbf{nk}-\mathbf{q_f}|^4}\mathrm{Tr}\left\{\Gamma_n\frac{(1+\lambda_i\gamma_5\not{s}_i)}{2}(c\not{p}_i+c^2)\overline{\Gamma}_n\frac{(1+\lambda_f\gamma_5\not{s}_f)}{2}(c\not{p}_f+c^2)\right\} \tag{2.10}$$

In Eqs (2.9) and (2.10)

$$\overline{\Gamma}_n = \gamma^0\Gamma_n^\dagger\gamma^0 \tag{2.11}$$

Using REDUCE[12], one finds that the polarized DCS

$$\frac{d\sigma}{d\Omega_f}(\lambda_i,\lambda_f) = \sum_{n=-\infty}^{+\infty}\frac{Z^2|\mathbf{q_f}|}{c^4|\mathbf{q_i}|}\frac{(\mathcal{A}_\backslash + \lambda_\rangle\lambda_\{\mathcal{B}_\backslash)}{|\mathbf{q_i}+\mathbf{nk}-\mathbf{q_f}|^4} \tag{2.12}$$

where $\mathcal{A}_\backslash$ and $\mathcal{B}_\backslash$ are given by

$$\begin{aligned}\mathcal{A}_\backslash = {} & A_1B_{0n}^2 + B_1B_{1n}^2 + C_1B_{2n}^2 + D_1B_{3n}^2 + E_1B_{0n}B_{1n} + F_1B_{0n}B_{2n} + G_1B_{0n}B_{3n} \\ & + H_1B_{1n}B_{2n} + X_1B_{1n}B_{3n} + Y_1B_{2n}B_{3n},\end{aligned} \tag{2.13}$$

$$\begin{aligned}\mathcal{B}_\backslash = {} & A_2B_{0n}^2 + B_2B_{1n}^2 + C_2B_{2n}^2 + D_2B_{3n}^2 + E_2B_{0n}B_{1n} + F_2B_{0n}B_{2n} + G_2B_{0n}B_{3n} \\ & + H_2B_{1n}B_{2n} + X_2B_{1n}B_{3n} + Y_2B_{2n}B_{3n},\end{aligned} \tag{2.14}$$

These coefficients are very lengthy see[10]. So, the two polarized DCS (helicity non flip ($\lambda_f = \lambda_i = 1$) and helicity flip ($\lambda_f = -\lambda_i = 1$)) are given by

$$\left(\frac{d\sigma}{d\Omega_f}\right)_{(\lambda_f=\pm\lambda_i=1)} = \sum_{n=-\infty}^{+\infty}\frac{Z^2|\mathbf{q_f}|}{c^4|\mathbf{q_i}|}\frac{(\mathcal{A}_\backslash \pm \mathcal{B}_\backslash)}{|\mathbf{q_i}+\mathbf{nk}-\mathbf{q_f}|^4} \tag{2.15}$$

Therefore, we obtain for the degree of polarization

$$P = \frac{(\frac{d\sigma}{d\Omega_f})_{non\ flip} - (\frac{d\sigma}{d\Omega_f})_{flip}}{(\frac{d\sigma}{d\Omega_f})_{non\ flip} + (\frac{d\sigma}{d\Omega_f})_{flip}} \tag{2.16}$$

It is easy to show both analytically and numerically that the unpolarized DCS given in (2.9) is such that

$$\frac{d\overline{\sigma}}{d\Omega_f} = \left(\frac{d\sigma}{d\Omega_f}\right)_{non\ flip} + \left(\frac{d\sigma}{d\Omega_f}\right)_{flip} \tag{2.17}$$

III. RESULTS AND DISCUSSIONS

A. The non relativistic regime

In this regime, one expects that the dressing of angular coordinates of $\mathbf{p_i}$ and $\mathbf{q_i}$ as well as those of $\mathbf{p_f}$ and $\mathbf{q_f}$ will not be important and this is indeed the case since $\cos(\mathbf{p_i}, \mathbf{p_f}) \simeq \cos(\mathbf{q_i}, \mathbf{q_f})$. While the helicity non flip polarized differential cross section (in short DCS (↑)) and the helicity flip polarized differential cross section (DCS (↓)) are influenced by the number of photons exchanged, it is not the case for the degree of polarization which remains nearly constant until the cut off is reached. We recall that when the cut off is reached, the various DCSs do not vary anymore since the arguments of the ordinary Bessel functions become close to their indices[7]. This situation is shown in Fig. 1 where we give the three DCSs : DCS (↑), DCS (↓) and the unpolarized DCS. For every simulation and for any number of photons exchanged, the sum of DCS (↑) and DCS (↓) always gives the unpolarized DCS.

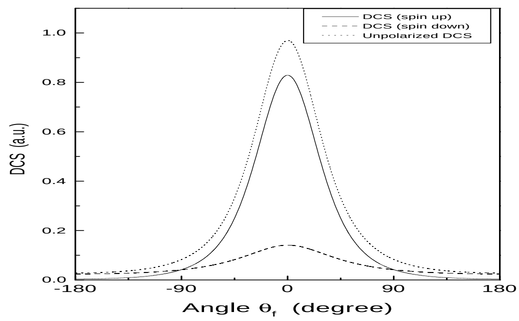

FIG. 1. The three DCSs : DCS (↑), DCS (↓) and unpolarized DCS scaled in 10^{-4} as functions of the angle θ_f in degree for $n = \pm 100$.

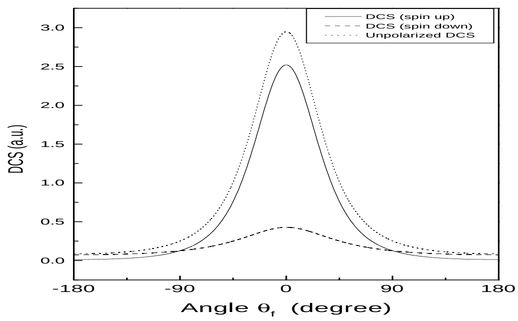

FIG. 2. The three DCSs : DCS (↑), DCS (↓) and unpolarized DCS scaled in 10^{-4} as functions of the angle θ_f in degree for an exchange of ±300 photons.

When the number of photons exchanged reaches the cut off, which in the non relativistic regime is ±300, the three DCSs increase as shown in Fig. 2 but the degree of polarization increases very slowly with respect to the number n of photons exchanged.

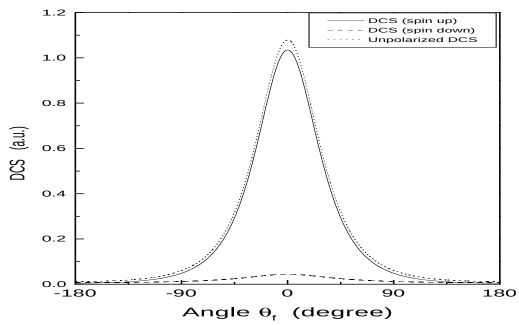

FIG. 3. The three DCSs : DCS (↑), DCS (↓) and unpolarized DCS scaled in 10^{-11} as functions of the angle θ_f in degree for an exchange of ±100 photons in the relativistic regime $\gamma = 2.0$, $E = 1.0$

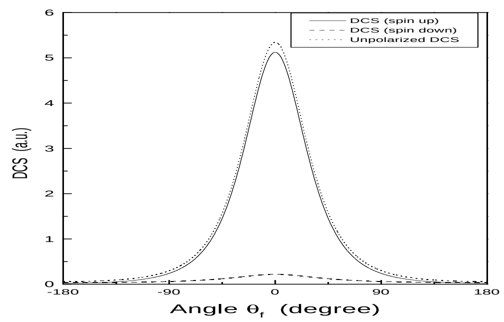

FIG. 4. The three DCSs : DCS (↑), DCS (↓) and unpolarized DCS scaled in 10^{-11} as functions of the angle θ_f for an exchange of ±500 photons in the relativistic regime $\gamma = 2.0$, $E = 1.0$

It is still close to $cos(\theta_{if})$ and increasing the number of photons has no influence on its value. If such a behavior holds for the relativistic regime, then at least to first order in perturbation theory, we can draw the following conclusions : while the various DCSs are reduced by the influence of the laser field with increasing values of the field strength as well as the incoming electron relativistic kinetic energies, the corresponding degrees of polarization are weakly sensitive to the presence of the laser field or in other terms to the number of photons exchanged.

B. The relativistic regime

In this regime, the cut off is ± 83000 photons that can be exchanged with the laser field and the corresponding DCSs are reduced. They are of the order 10^{-11} (for the laser free case they are of the order 10^{-8}) and we show in Fig. 3 the behavior of the three DCS. As the energy of the incident electron is increased (and also the electric field strength), the helicity flip polarized differential cross section DCS ($\downarrow$) is almost negligible, showing that the leading process in this regime is likely to favor the helicity non flip polarized differential cross section DCS ($\uparrow$). For an exchange of ± 100 photons and for an exchange of ± 500 photons, the situation remains the same as to the relative importance of the two DCSs. The increase in the value of the DCS ($\uparrow$) is simply due to the fact that we have summed over $n = \pm 500$ photons and this can be shown in Fig. 4. On the other hand, there is a small change in the degree of polarization even if we sum over a much larger number of photons. This is due to the fact that the degree of polarization P can be written as

$$P = 1 - 2\frac{\frac{d\sigma(\downarrow)}{d\Omega}}{\frac{d\sigma(\uparrow)}{d\Omega} + \frac{d\sigma(\downarrow)}{d\Omega}} \tag{3.1}$$

and that the second term in the above equation is weakly-dependent with respect to the number of photons exchanged. Once again, one can not distinguish between the degree of polarization corresponding to an exchange of ± 100 photons and $n = \pm 500$ photons.

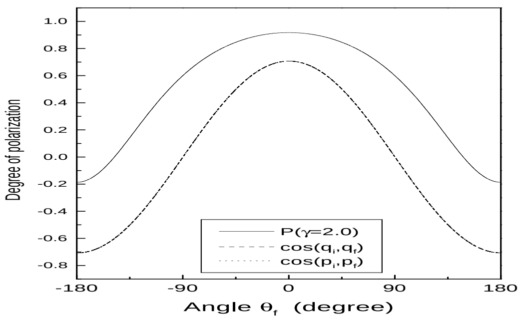

FIG. 5. The behaviors of the degree of polarization $P(\gamma = 2.0)$, $cos(\mathbf{q_i}, \mathbf{q_f})$ and $cos(\mathbf{p_i}, \mathbf{p_f})$ as functions of the angle θ_f. The curves corresponding to $\cos(\mathbf{p_i}, \mathbf{p_f})$ and $\cos(\mathbf{q_i}, \mathbf{q_f})$ overlap.

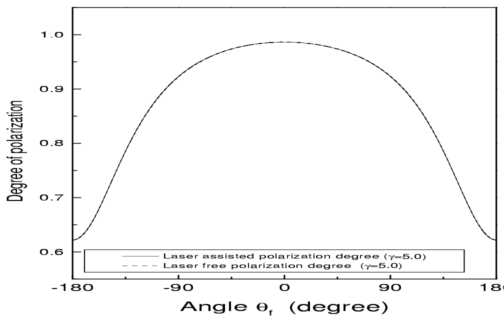

FIG. 6. The behaviors of the degree of polarization $P(\gamma = 5.0, \quad n = \pm 1000)$ and the laser free degree of polarization $P(\gamma = 5.0)$ as functions of the angle θ_f. The two curves overlap.

This degree of polarization is shown in Fig. 5. At this stage, an important point needs to be clarified. Indeed, one may ask if the quasi-independence of the degree of polarization with respect to the number of photons exchanged is an approximate or an exact result. the answer is approximate, since we have been checked for various geometries leading always to the same aforementioned conclusion.

C. Conclusion

In this work, we have studied the behavior of the three DCSs (the helicity non flip polarized differential cross section, the helicity flip polarized differential cross section and the unpolarized differential cross section) in the presence of a linearly polarized laser field. We have mainly studied the non relativistic and the relativistic regime and in all cases, the sum of the two polarized differential cross sections always gives the unpolarized differential cross section while the degree of polarization is independent with respect to the number of photons exchanged and is very close to the laser free degree of polarization. These results have been obtained in the first order of perturbation theory and are valid for a very wide range of angular geometries.

IV. REFERENCES

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