Algorithm for extracting infinite clusters support of mechanical and electrical properties in percolation networks

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The concept of the percolation is a rather intuitive simple model for the physical problems of very different nature which appear in strongly inhomogeneous mediums. Being a phenomenon in threshold, it represents an some interest practises in many theoretical and industrial applications, so that a good comprehension of the phenomena of percolation becomes extremely useful, the interest of a reliable algorithm, economic and rapid allowing the search for percolants structures in the percolation of bonds. In our laboratory, we re-examined a method for extraction of the infinite cluster. Method based on sweeping and the treatment row by row of the structure. This will enable us to accelerate the evaluation of conductivity as well as the currents distribution of a network formed by a random distribution of electric resistances.

Key words: percolation, percolation threshold, clusters distribution, labelling technique, random resistor networks, and infinite set of exponents.

1. INTRODUCTION

The study of physical systems with critical phenomena, has gained interest these last decades [1-7]. It is well established that systems, with random structure, have properties where scaling laws lead to exponents depending strongly on the dimension of the space; and where the microscopic details are irrelevant [8]. For example, we can cite the diffusion in disordered media [9-11]; random walk on percolation clusters [12]; percolation in continuum [22]; electrical transport on a random resistor network [13-17], or fractures in materials [18].

In these systems, the structures [19; 29; 30] and distributions [20; 21; 32] of clusters play a pertinent role. The systems at the percolation threshold exhibit fractal and multifractal properties [22-27]. Therefore the extracting of the infinite cluster (support of electrical currents) is necessary for computing all physical parameters of structures. In fact, if we don't cut out the finite clusters, we obtain a system of algebraic equations without solution. For instance it's a must the algorithm of the extracting percolation cluster.

Basic concepts in percolation theory

The objective of this section was briefly to present some of the basic concepts most relevant in the model of percolation.

The older paper introducing the percolation has been published by Broadbent and Hammersley in 1957 [31]. The percolation plays a fundamental role in a considerable number of physical phenomena in which the disorder is present within the medium[38]. *The percolation term* comes from *percolation* who meant filtration.Used in a great number of situations, it evokes the concepts of propagation and agglutination in the random mediums partially inter-connected.A great number of phenomenon's can be described by approaches of percolation, such the transitions [34]:

- composite matériel conductor -insulator
- composite material conductor - superconductor
- magnetic transformation
- the manufacture of the coffee in a percolator
- propagation of fires of forests

In this description of the concept of percolation, one finds fundamental assumptions [39]:

- The studied phenomenon must take seat in space containing a great number of element;
- The relation between the elements rests on a *local* aspect like the physical adjacency or another dimension of proximity;
- This relation between the elements with a random character.

From these assumptions, the theory of the percolation describes the appearance of a critical phenomenon at the *total* level: to the lower part of the threshold percolation the information is limited to the small island where it was initiated, whereas with the top of the *threshold* it through the studied medium [40].

There are two types of percolation:percolation of sites and that of bonds:

- The model of the percolation of sites considers that all the bonds are active but hat the contacts between the bonds are controlled by the sites in a random state.
- The model of percolation of bonds postulates that all the sites are active, but that the contacts between sites are controlled by the bonds with random state [37].

An interesting example is that of a square network of random resistances (RRN).

Let us consider a square network of bond inclined to 45° of size L, and cut a fraction (1-p) of the bonds of this network chosen randomly, with p a given probability.When p is near to 1, only some holes will have been creates.On the contrary when p is near to 0, the whole of the network is parcelled out in small cluster.Between these two extremes, there is a particular value of p, named *threshold of percolation*, and noted p_c such as for $p \geq p_c$ exists a cluster of infinite extension, and for $p < p_c$ remain only of the clusters of finished size (fig3).

The physical phenomenon brought into play is then a true transition from second-order phase:

- The parameter of order is the probability to belong to the infinite cluster p_∞.
- The parameter of control of this transition is the fraction p.

In the theory of percolation, the existence of the threshold is fundamental.This critical value is characterized by [37]:

$$P(p)\begin{cases} = 0 & \text{si} \quad p \prec p_c \\ \succ 0 & \text{si} \quad p \geq p_c \end{cases}$$

The threshold of percolation p_c is defined then as the first value of p where the probability of percolation is not null.In the models of percolation of sites and bonds, one intuitively obtains the following properties [36]:

- The probability of percolation is null if the rate activity ($p = 0$) is null:*p (0)=0*;
- The probability of percolation is certain if the rate activity ($p = 1$) is 100%:*p(1)=1*;
- The probability of percolation is a no decreasing function of *p*.
  $p \to P(p) > 0$ .Consequently, the general form of the function of probability of percolation $P(p)$ is supposed move according to the diagram of the figure 1.

With the threshold percolation, there is thus a cluster of particles which extends through all the network.The structure of the clusters of percolation is extremely tortuous.To describe these clusters, the physicists were interested in the "*backbone*", true skeleton of the infinite cluster. The *backbone* is the support of the shortest ways connecting the two ends of the network.

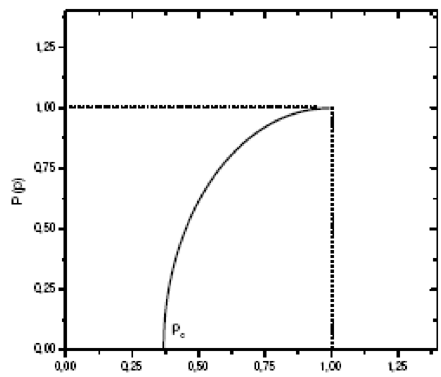

FIG1: Probability of percolation $P(p)$

The image which one can have of a cluster of percolation close to the threshold is as follows:

- The infinite cluster is composed of *a backbone* on which branches are grafted, not taking part in conduction, named *dead arms*;
- The backbone itself is decomposable in new subset:
- *Significant bonds*, such as the cut of only one bond stops the current in the system (all the intensity is reduced in these bonds which known as is simply connected);
- Bonds which are multi-connected and which constitute *the blobs* of the infinite cluster.

In this study we then propose an algorithm of search for infinite cluster on square networks preablement degraded.

2. ALGORITHM OF RESEARCH OF THE INFINITE CLUSTER

Model of simulation

Our numeric treatment has been established on a random resistance network that is presented with the following features:

- It is a square network inclined to 45° that contain L sites on every side. The slope of the network makes that the number of bonds of the network N is equal to L'*L' with $L' = 2L$. For avoid confusions with the right network, we will say a network of size L '.
- A tension is applied between two edges of the network and a condition of cyclicityto the two other edges

We adopted an inclined network bus in the case of a study of the currents distribution in a right network (non-inclined) and regular, subjected to a potential difference, the currents are on only in the bonds which are directed in the direction of the decrease of the potential.A bond crosses being on equipotential, will not see any current crossing it.The network is degraded with a rate $1-p$.In other words, we cut $(1-p)N$ bonds randomly (they are bonds which does not make pass from current).The resistance network then arises as follows: it is a network whose resistances take two values.They have a resistance unit with a probability p, or an infinite resistance with a probability $1-p$.

Algorithm

In our work, we developed a dynamic method of analysis of the distribution of the percolant cluster on structures.Indeed, this one uses variables created progressively with the needs for the program.In addition a technique of sweeping of the network by line was developed [37]. allowing to accelerate the algorithm of Kopelman [4].

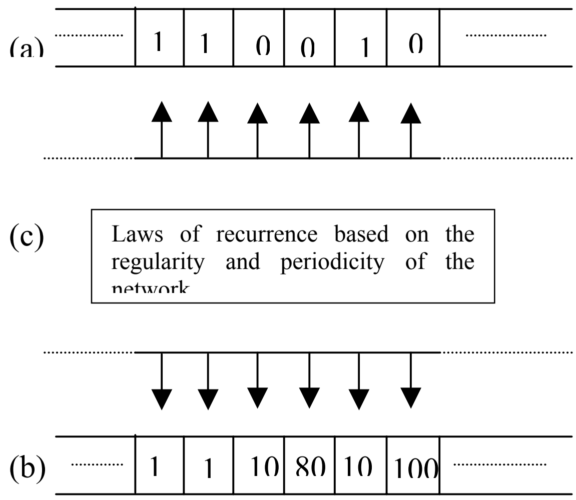

FIG2 - description of the "window":(a) represents the state of the bonds of the network in a window, (b) the membership of the various clusters of the preceding bonds, (c) rule of correspondence between (a) and (b) [37].

Also this method offers the double advantage, on the one hand to optimize the place memory what makes it possible to treat great structures, in addition, to reduce the computing times, as we announced higher, by the examination of a part reduced, through a "window", of the network (fig2).The algorithm using this "window" breaks up as follows: Having opened a *window* on a line of bonds, of degraded network, we examine their state and we connect them to the state bonds which are right front. Like between them.This technique is relatively easy if one exploits the properties of regularity and periodicity of the networks considered.
The figure3 is extracted thanks to this program.

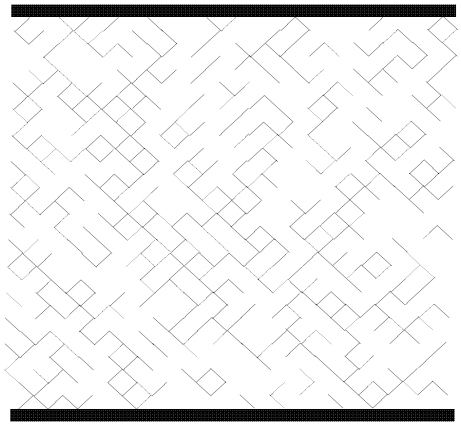

FIG3:Cluster of percolation

CONCLUSION

The development of our algorithm was constantly guided by a concern of effectiveness and speed of calculation but also by a preoccupation with a saving in memory and time, thus leading to a code which can be used on any class of computers without any particular specificity.It would be currently interest in to work out an algorithm, based on the same principles and making it possible to obtain the skeleton of the infinite cluster. This more delicate and more difficult procedure will make it possible to study the various properties of transport on structures percolantes of very big sizes.

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